Solvay Brussels School Economics & Management

ULB

Financial Risk Management and Governance

Credit Risk (review) (individual credit risk)

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This is a review (from your previous courses)

Ratings ⟷ historical data ⟷ determinants of default risk (D/V, ICR)

models

ⓐ TOP-DOWN : reduced-form / probabilistic } models.

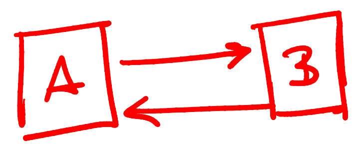
ⓑ BOTTOM-UP : structural models (corporate finance)

of course, you can manage credit risks

→ guarantees, collateral

→ netting ⟶ look at ISDA for example.

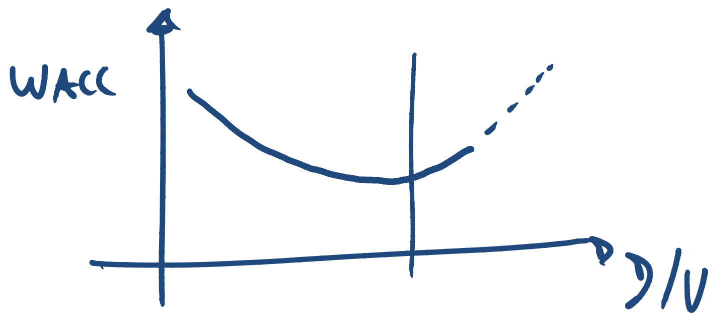
→ credit derivatives



Understanding what credit risk is…

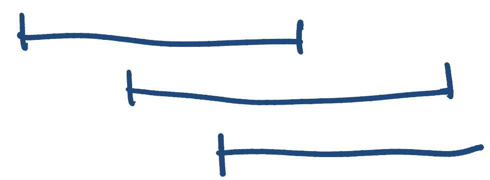
Motivations

- In the WACC, we need to know
  - » How /why kd can adjust as D/V increases?
  - » What is the risk premia about?



- BUT: How is this risk comparable to a standard market risk? ≠ Market risk
  - » This risk implies a discontinuity in time… ← talking about default risk.
  - » Estimation: Survivorship bias → panel analysis of survivors

Y1 Y2 Y3
100
10
65



How could we come up with a value for this risk premia?

- Reduced-form approaches
- Structural models

A potential agenda...

- Credit risk in general in Asset Pricing
  - Reduced-form vs structural models
  - Pricing a single bond
    - ✓ Merton(74,77): 0-coupon bond
    - ✓ Leland(94): coupon-bearing bond
  - Pricing of bond portfolios
  - Credit risk in derivatives
- Corporate Credit Risk
  - Structural default vs. Cash-flow insolvency
  - Ratings/Monitoring
  - WACC & Optimal capital structure problems
  - Capital allocation inter-corporate and intra-corporates
- Sovereign Credit Risk

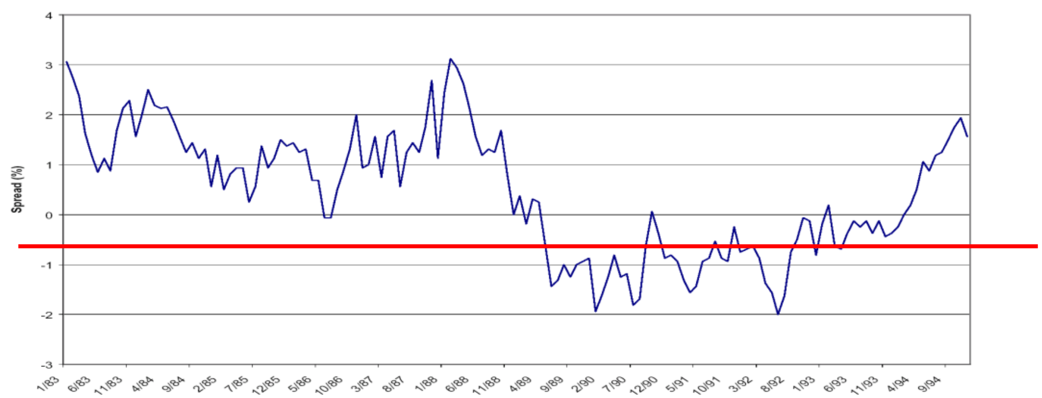
  + Firm or Country growth linked to debt levels
    - Impact of sanctions/Loss of reputation/Cuts in production or exports
- Integration of Market and Credit Risks → Portfolio Management
- Regulatory rules: Basle II Accord

What is credit risk?

- Credit risk existence derives from the possibility for a borrower to default on its obligations to pay interest or to repay the principal amount.
  - » As valued today...
  - » We are valuing today a discontinuity in the future that may potentially happen but maybe not...
- Consequence:
  - » Cost of borrowing > Risk-free rate
  - » Spread = Cost of borrowing – Risk-free rate (usually expressed in basis points)
  - » Volume
  - » Rating change
    - ✓ Internal (for loans)
    - ✓ External: rating agencies (for bonds)

What is credit risk? (2)

- ≠ Market risk
  - » Survivorship bias → panel analysis of survivors
- The potentiality of a default of a counterpart
  - » Default time/point
  - » Evolution to default



- Continuous or not?
  - » Continuity provides a parallel framework to those existing for market risks
  - » But the event itself is better explained as a "jump" to default at some point in the future, with some "magnitude"
  - » But we can look at the evolution of the creditworthiness of the firm and examine it as a continuous process than may have "jumps".

Ratings & rating agencies

- The traditional practice is to « rate » issuers and issuances...
  - Moody's (www.moodys.com)
  - Standard and Poors (www.standardandpoors.com)
  - Fitch/IBCA (www.fitchibca.com)
- Letter grades (qualitative score) to reflect safety of bond issue

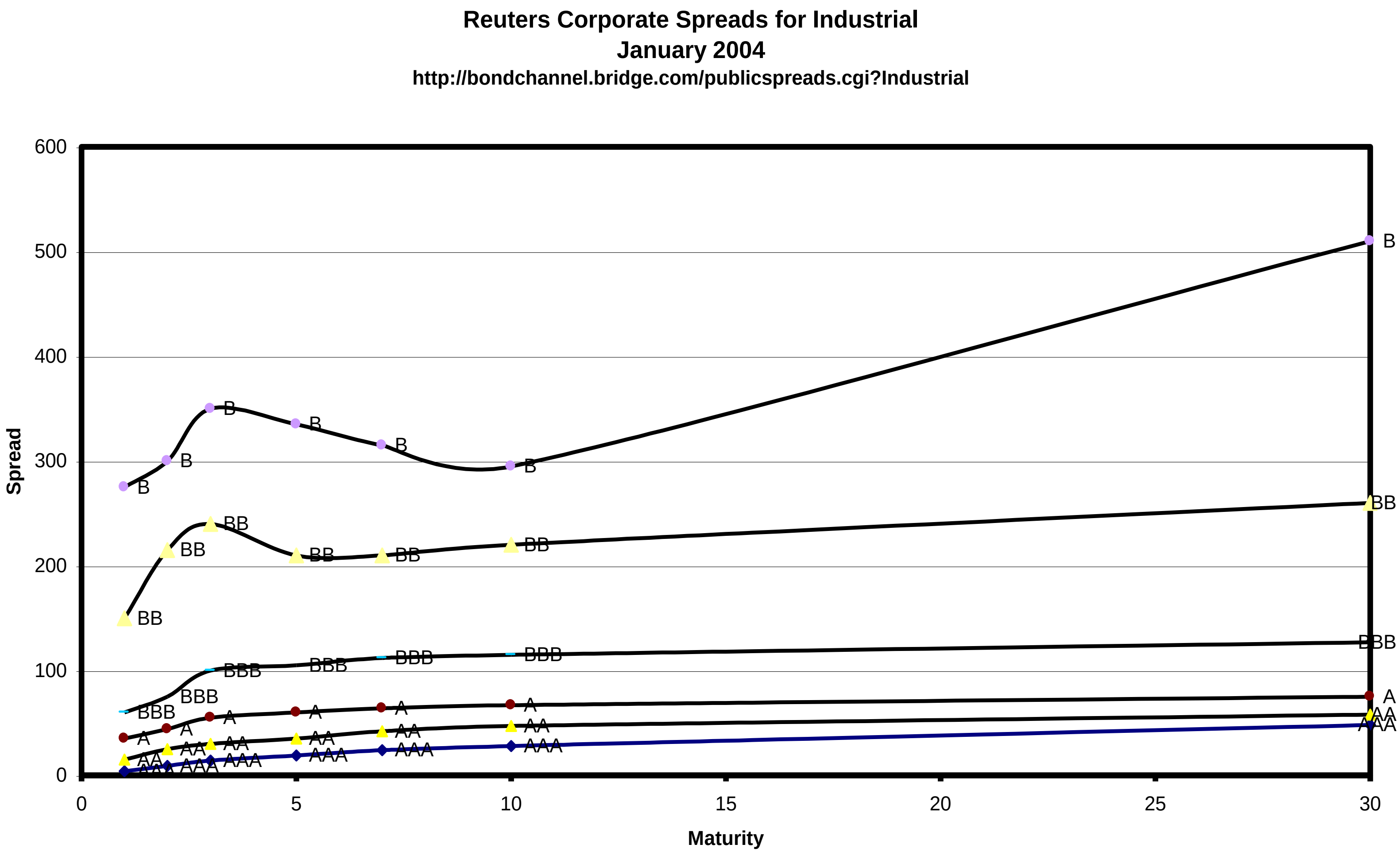
Long-term	S&P	Moody's
	AAA	Aaa
	AA	Aa
	A	A
	BBB	Baa
	BB	Ba
	B	B
	CCC	Caa
	CC	Ca
	C	C
	CI,R,SD,D	WR,P

NR = non-rated

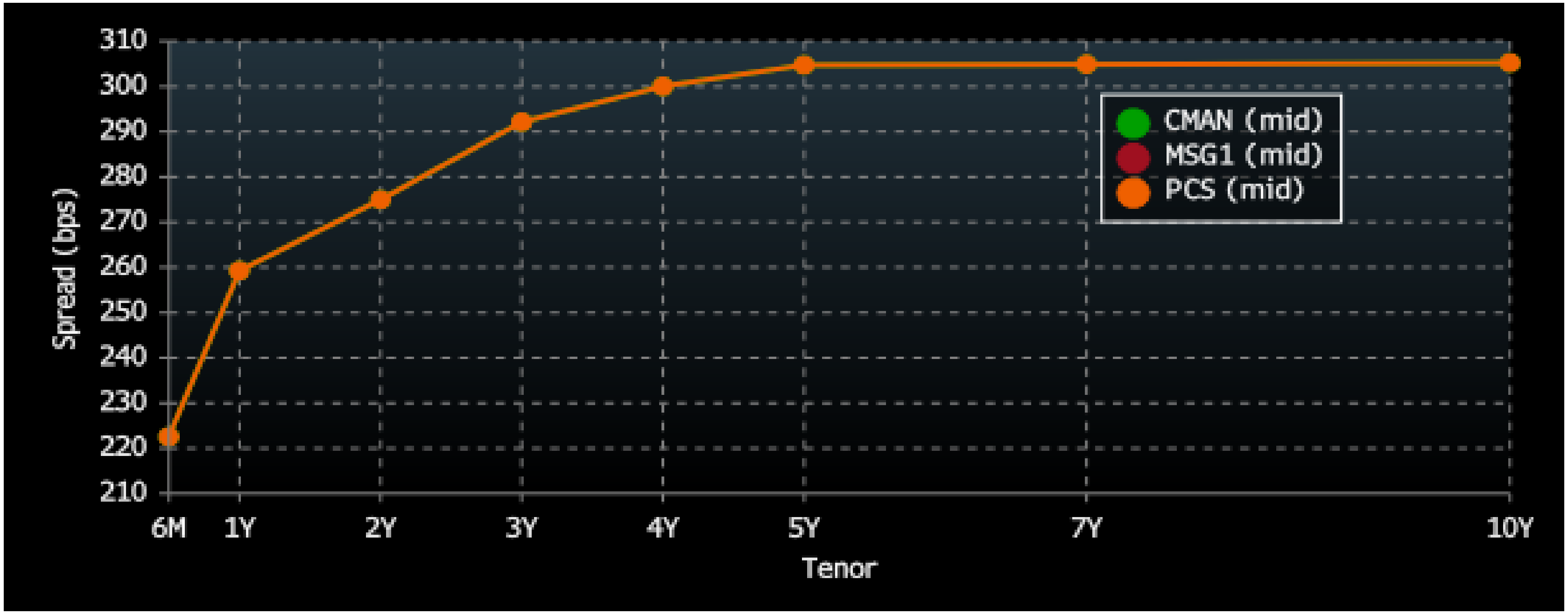
Short-term	S&P
	A-1
	A-2
	A-3
	B
	C
	D

Moody's
P-1
P-2
P-3
NP
A,B,C,D,E for banks

Credit Spreads by rating class

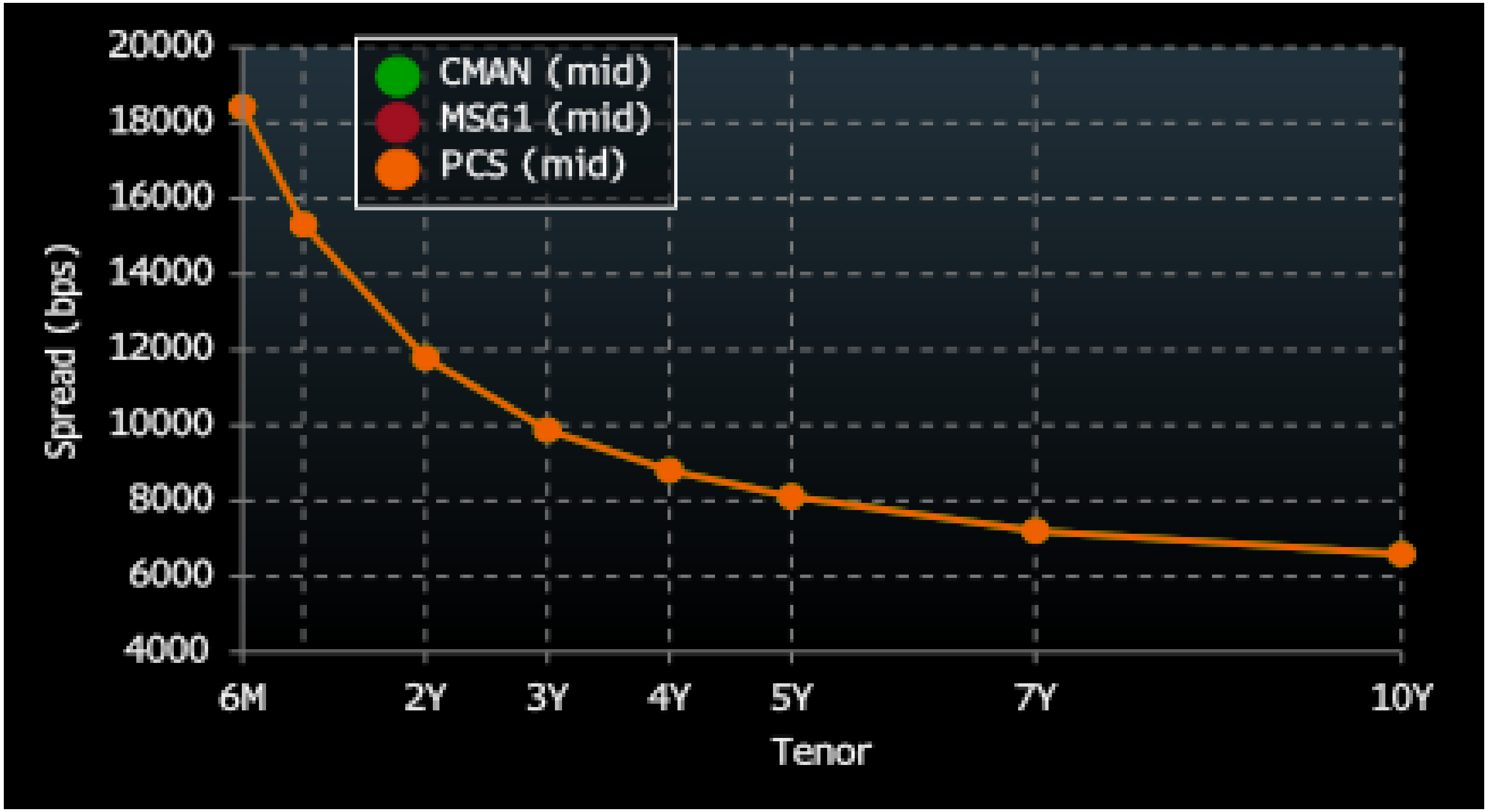


Belgium CDS by term



Source: Bloomberg, Nov 30th, 2011

Greece CDS by term



Source: Bloomberg, Nov 30th, 2011

Determinants of Bonds Safety

- Key financial ratio used:
  - Coverage ratio: EBIT/(Interest + lease & sinking fund payments)
  - Leverage ratio
  - Liquidity ratios
  - Profitability ratios
  - Cash flow-to-debt ratio
- Rating Classes and Median Financial Ratios, 1997-1999

Rating Category	Coverage Ratio	Cash Flow to Debt %	Return on Capital %	LT Debt to Capital %
AAA	17.5	55.4	28.2	15.2
AA	10.8	24.6	22.9	26.4
A	6.8	15.6	19.9	32.5
BBB	3.9	6.6	14.0	41.0
BB	2.3	1.9	11.7	55.8
B	1.0	(4.6)	7.2	70.7

Source: Bodies, Kane, Marcus 2002 Table 14.3

Inputs...

- Used in probabilistic models and integrated in the regulation:
  - » PD: probability of default
  - » LGD: loss-given-default (may be in % or in value)
  - » EAD: exposure-at-default (used by Basle II to separate the LGD in % from the real exposure beard by the firm).

PD × LGD × EAD

2% 10 mio EUR ~~100%~~

50% thanks to a mortgage on 5 mio

5 mio EUR

EL (expected loss)

Default Rate Calculation

- Incorrect method:
  - » Number defaults/Total number of bonds
    - ✓ Ignores growth/reduction of bond market over time
    - ✓ Ignores aging effect: takes time to get into trouble
- Correct method: cohort style analysis
  - » Pick up a cohort
  - » Follow it through time

➔ Survivorship bias...

Transition matrix of rating migrations

Exhibit 15 - Average One-Year Letter Rating Migration Rates, 1920-2007*

Cohort Rating	End-of-Period Rating: Aaa	Aa	A	Baa	Ba	B	Caa	Ca-C	Default	WR
Aaa	87.292	7.474	0.841	0.167	0.024	0.001	0.000	0.000	0.000	4.200
Aa	1.261	85.204	6.465	0.687	0.175	0.037	0.002	0.004	0.063	6.103
A	0.081	2.934	85.086	5.298	0.693	0.108	0.019	0.008	0.076	5.696
Baa	0.042	0.293	4.618	81.140	5.107	0.776	0.150	0.016	0.293	7.565
Ba	0.007	0.082	0.476	5.917	73.643	6.977	0.557	0.051	1.324	10.967
B	0.007	0.054	0.173	0.630	6.292	71.459	5.011	0.502	3.917	11.955
Caa	0.000	0.028	0.037	0.216	0.906	8.920	62.797	3.549	12.000	11.548
Ca-C	0.000	0.000	0.116	0.000	0.474	3.240	7.698	55.323	19.872	13.277

** Monthly cohort frequency*

Withdrawn.

T

Source: Moody's, Corporate Default and Recovery Rates, 1920-2007, February 2008.

Rating matrix after "n" years : T^n after "renormalizing T"

Cumulative default rates

Exhibit 26 - Average Cumulative Issuer-Weighted Global Default Rates, 1920-2007*

Rating	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Aaa	0	0	0.019	0.077	0.163	0.255	0.368	0.531	0.701	0.897
Aa	0.061	0.181	0.286	0.446	0.704	1.013	1.336	1.651	1.953	2.294
A	0.073	0.237	0.5	0.808	1.116	1.448	1.796	2.131	2.504	2.901
Baa	0.288	0.85	1.561	2.335	3.142	3.939	4.707	5.475	6.278	7.061
Ba	1.336	3.2	5.315	7.49	9.587	11.56	13.363	15.111	16.733	18.435
B	4.047	8.786	13.494	17.72	21.425	24.656	27.594	30.037	32.154	33.929
Caa-C	13.728	22.46	29.029	33.916	37.638	40.584	42.872	44.921	46.996	48.981
Investment-Grade	0.144	0.431	0.805	1.23	1.687	2.157	2.626	3.091	3.578	4.076
Speculative-Grade	3.59	7.237	10.752	13.919	16.714	19.179	21.372	23.336	25.114	26.827
All Rated	1.406	2.878	4.315	5.626	6.802	7.854	8.803	9.667	10.484	11.281

** Includes bond and loan issuers rated as of January 1 of each year.*

Source: Moody’s, Corporate Default and Recovery Rates, 1920-2007, February 2008.

Default rates by industry group

Exhibit 35 - Annual Default Rates by Broad Industry Group, 1970-2007

Year	Banking	Capital Industries	Consumer Industries	Energy & Environment	FIRE	Media & Publishing	Retail & Distribution	Sovereign & Public Finance	Technology	Transportation	Utilities
1970		0.000	0.922	0.000	20.000	0.000	0.000	0.000	0.840	16.107	0.000
1971		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.400	0.000
1972		0.355	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.226	0.000
1973		0.352	0.000	0.000	0.000	0.000	2.899	0.000	0.000	1.667	0.000
1974		0.354	0.000	0.000	0.000	0.000	2.985	0.000	0.000	0.000	0.000
1975	0.000	0.356	0.769	0.000	0.000	4.444	1.504	0.000	0.000	0.000	0.000
1976	0.000	0.353	0.725	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1977	0.000	0.000	0.738	0.000	0.000	4.167	0.000	0.000	0.000	1.810	0.000
1978	0.000	0.000	0.738	1.227	0.000	0.000	1.538	0.000	0.735	0.000	0.000
1979	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.719	0.000	0.000
1980	0.000	0.743	0.000	1.124	0.000	0.000	0.000	0.000	0.000	0.957	0.000
1981	0.000	0.362	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.966	0.000
1982	0.000	1.091	0.000	0.926	0.000	3.922	4.545	0.000	1.869	2.062	0.000
1983	0.000	1.064	0.563	2.449	0.000	0.000	0.000	0.000	0.615	4.020	0.408
1984	0.000	0.697	1.061	3.953	0.000	0.000	0.000	0.000	1.813	1.058	0.000
1985	0.000	1.499	1.351	3.425	1.117	0.000	0.000	0.000	0.560	0.000	0.000
1986	0.000	3.315	1.938	7.971	0.000	1.802	0.962	0.000	0.517	2.778	0.000
1987	0.399	2.368	2.393	4.895	0.000	1.266	1.646	0.000	0.472	0.000	0.813
1988	2.034	0.781	2.548	1.434	0.583	3.315	1.550	0.000	1.210	0.000	0.413
1989	2.128	2.914	4.088	0.000	3.200	6.486	0.709	16.667	1.186	1.843	0.000
1990	2.677	5.148	7.837	0.649	0.000	5.882	7.213	0.000	1.188	5.479	0.402
1991	1.813	3.547	3.663	1.290	0.484	4.000	9.353	0.000	1.590	8.911	0.815
1992	0.503	1.918	2.756	0.639	0.459	7.042	2.362	0.000	1.139	0.000	0.813
1993	0.469	1.515	1.119	1.170	0.000	2.759	2.290	0.000	0.367	0.000	0.000
1994	0.000	0.202	0.910	0.000	0.000	1.183	2.516	0.000	1.042	2.553	0.388
1995	0.000	1.221	2.663	0.488	1.064	0.000	1.729	0.000	0.649	0.826	0.000
1996	0.000	0.488	1.245	0.885	0.000	2.381	0.560	0.000	0.596	0.000	0.363
1997	0.000	0.438	2.191	0.000	0.271	1.303	2.564	0.000	0.543	0.766	0.000
1998	0.131	1.133	2.178	0.946	0.888	2.667	5.783	0.000	0.698	0.669	0.000
1999	0.251	2.211	4.489	4.545	0.600	2.746	2.637	3.448	1.858	5.573	0.630
2000	0.000	4.103	6.226	1.381	0.781	1.684	6.009	0.000	2.388	4.416	0.000
2001	0.122	7.025	5.518	1.628	1.167	3.805	7.745	0.000	7.295	3.145	0.569
2002	0.611	2.933	2.078	4.326	0.184	9.670	3.030	0.000	8.810	5.229	0.546
2003	0.000	2.579	1.975	1.550	0.352	3.526	4.124	0.000	4.095	2.632	0.543
2004	0.000	1.497	2.285	0.253	0.172	1.538	1.111	0.000	0.713	1.307	0.265
2005	0.112	1.321	0.500	0.742	0.132	0.488	1.729	0.000	0.235	3.185	0.256
2006	0.000	1.528	0.963	0.000	0.215	1.399	1.102	0.000	0.709	1.250	0.000
2007	0.000	0.838	0.643	0.000	0.000	0.911	1.648	0.000	0.231	0.000	0.000

Russian's default.

Source: Moody's, Corporate Default and Recovery Rates, 1920-2007, February 2008.

Recovery rates

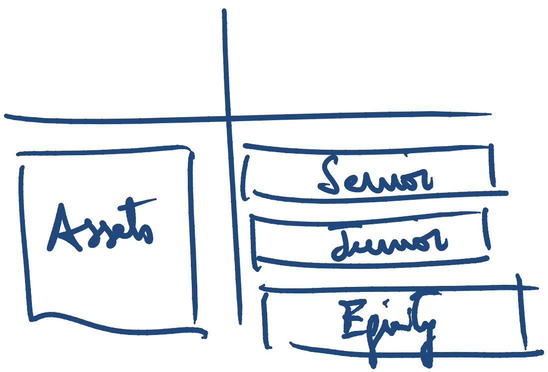
Exhibit 22 - Annual Average Defaulted Bond and Loan Recovery Rates, 1982-2007*

	Lien Position						
Year	Sr. Secured Bank Loans	Sr. Secured Bonds	Sr. Unsecured Bonds	Sr. Subordinated Bonds	Subordinated Bonds	Jr. Subordinated Bonds	All Bonds
1982	NA	$72.50	$35.79	$48.09	$29.99	NA	$35.57
1983	NA	$40.00	$52.72	$43.50	$40.54	NA	$43.64
1984	NA	NA	$49.41	$67.88	$44.26	NA	$45.49
1985	NA	$83.63	$60.16	$30.88	$39.42	$48.50	$43.66
1986	NA	$59.22	$52.60	$50.16	$42.58	NA	$48.38
1987	NA	$71.00	$62.73	$44.81	$46.89	NA	$50.48
1988	NA	$55.40	$45.24	$33.41	$33.77	$36.50	$38.98
1989	NA	$46.54	$43.81	$34.57	$26.36	$16.85	$32.31
1990	$75.25	$33.81	$37.01	$25.64	$19.09	$10.70	$25.50
1991	$74.67	$48.39	$36.66	$41.82	$24.42	$7.79	$35.53
1992	$61.13	$62.05	$49.19	$49.40	$38.04	$13.50	$45.89
1993	$53.40	NA	$37.13	$51.91	$44.15	NA	$43.08
1994	$67.59	$69.25	$53.73	$29.61	$38.23	NA	$45.57
1995	$75.44	$62.02	$47.60	$34.30	$41.54	NA	$43.28
1996	$88.23	$47.58	$62.75	$43.75	$22.60	NA	$41.54
1997	$78.75	$75.50	$56.10	$44.73	$35.96	$30.58	$49.39
1998	$51.40	$48.14	$41.63	$44.99	$18.19	$62.00	$39.65
1999	$75.82	$43.00	$38.04	$28.01	$35.64	NA	$34.33
2000	$68.32	$39.23	$23.81	$20.75	$31.86	$15.50	$25.18
2001	$66.16	$37.98	$21.45	$19.82	$15.94	$47.00	$22.21
2002	$58.80	$48.37	$29.69	$23.21	$24.51	NA	$30.18
2003	$73.43	$63.46	$41.87	$37.27	$12.31	NA	$40.69
2004	$87.74	$73.25	$54.25	$46.54	$94.00	NA	$59.12
2005	$82.07	$71.93	$54.88	$26.06	$51.25	NA	$55.97
2006	$76.02	$74.63	$55.02	$41.41	$56.11	NA	$55.02
2007**	$67.74	$80.54	$51.02	$54.47	NA	NA	$53.53

** Issuer-weighted, based on 30-day post-default market prices. Discounted debt excluded.*

*** Loan recoveries in 2007 are based on 5 loans from 2 issuers, one of the 5 loans is 2nd lien debt*

Source: Moody's, Corporate Default and Recovery Rates, 1920-2007, February 2008.



Recovery rates

Average Corporate Debt Recovery Rates measured by post-default trading prices, 1982-2007[1]

	Issuer-Weighted				Value-Weighted		
Lien Position	2007	2006	1982-2007	Lien Position	2007	2006	1982-2007
Bank Loans				Bank Loans			
Sr. Secured	67.74%	76.02%	70.47%	Sr. Secured	74.21%	68.38%	65.52%
Sr. Unsecured	--	--	54.02%	Sr. Unsecured	--	--	46.00%
Bonds				Bonds			
Sr. Secured	80.54%	74.63%	51.89%	Sr. Secured	81.68%	75.32%	54.21%
Sr. Unsecured[2]	51.02%	55.02%	36.69%	Sr. Unsecured	56.34%	69.99%	34.85%
Sr. Subordinated[3]	54.47%	41.41%	32.42%	Sr. Subordinated	67.68%	38.26%	29.80%
Subordinated	--	56.11%	31.19%	Subordinated	--	61.05%	27.58%
Jr. Subordinated	--	--	23.95%	Jr. Subordinated	--	--	16.79%
Pref. Stock[4]				Pref. Stock**			
Trust Pref.	--	7.12%	11.66%	Trust Pref.	--	7.12%	12.97%
Non-trust Pref.	--	6.75%	23.22%	Non-trust Pref.	--	11.63%	19.92%

1. Based on 30-day post-default market prices.
2. 10 issuers had trading prices on their senior unsecured bonds in 2007. One of them had an extremely low recovery rate of 0.32. Excluding this observation, the average issuer- and volume-weighted senior unsecured bond recovery rate would have been 56.65 and 56.95, respectively
3. 7 issuers had trading prices on their senior subordinated bonds in 2007. One of them had an extremely high recovery rate of 103. Excluding this observation, the average issuer- and volume-weighted senior subordinated bond recovery rate would have been 46.39 and 48.54, respectively
4. Only includes defaults on preferred stock that are associated or followed by a broader debt default. Average recovery rates for preferred stock covers the period of 1983-2007.

Source: Moody's, Corporate Default and Recovery Rates, 1920-2007, February 2008.

Recovery rates... and their volatility

- A prior study

Class of Debt	Recovery Rate	Standard Deviation
Senior Secured Bank	47.54%	21.33%
Equipment Trust	65.93%	28.55%
Senior Secured Public	55.15%	24.31%
Senior Unsecured Public	51.31%	26.30%
Senior Subordinated Public	39.05%	24.39%
Subordinated Public	31.66%	20.58%
Junior Subordinated Public	20.39%	15.36%
All Subordinated Public	34.12%	20.35%
All Public	45.02%	26.37%

[0%, 100%]

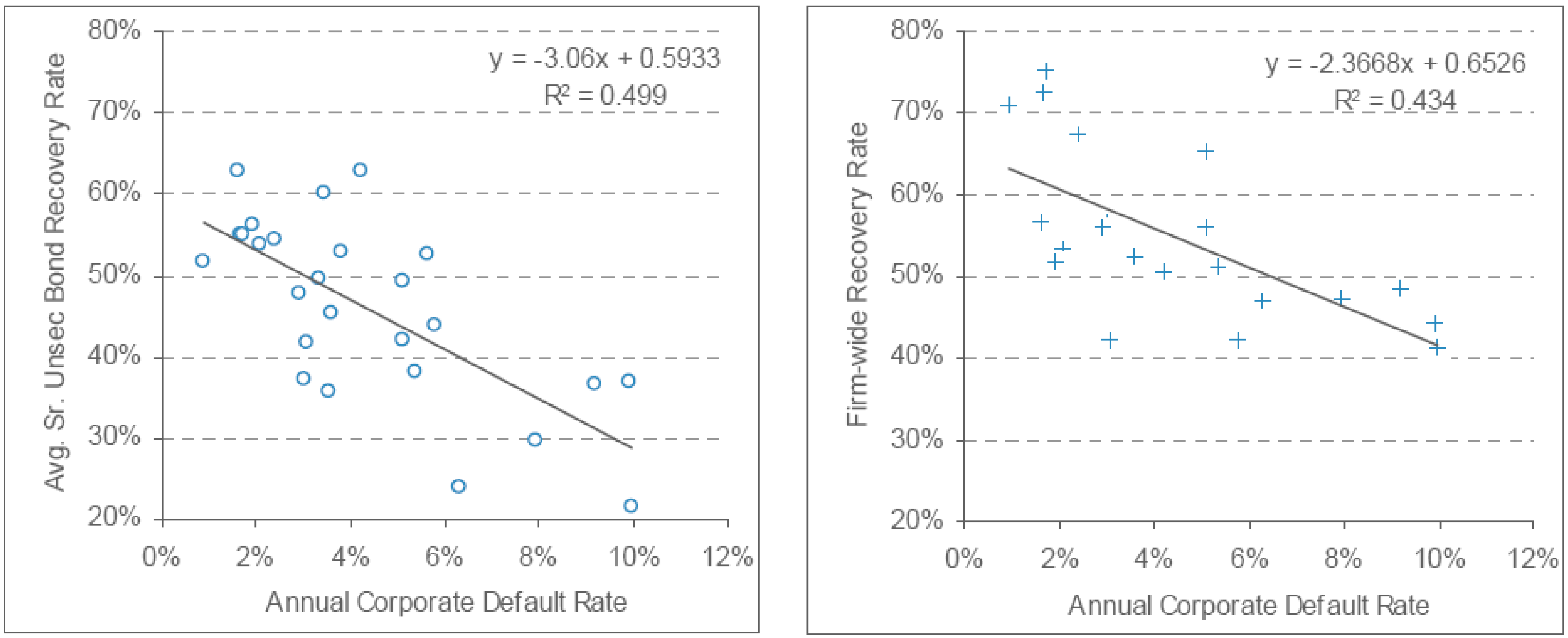
Basle's default : 40%

Correlation

Exhibit 10 – Correlation between Default and Recovery Rates, 1982-2007

Panel A

Panel B



Source: Moody's, Corporate Default and Recovery Rates, 1920-2007, February 2008.

What can we do about credit risk?

- Try to mitigate it (at the source)
  - Collateralisation
  - Guarantees
  - Covenants
- Price it
  - Various models
- Hedge it/Share it
  - Securitise
  - Insure

- Let’s try to price/value it!

Trying to quantify credit risk…

How do we try to quantify credit risk?

1) Historical stats
   - probabilities of default (PD)
   - recovery rates (R) or loss-given-default (1-R)
2) Scoring
   - Z-scores (Altman)
   - Ratings (Moody's, S&P, Fitch): PIT and TTC
3) Model credit spreads
   - An exchange rate (Jarrow, Jarrow & Turnbull)
   - Reduced-form models (Duffie & Singleton, Lando)
     - ✓ Calibration of PD and LGD to traded products
   - Through the option pricing model (Merton)
   - Strategic default (Anderson & Sundaresan)
4) Portfolio credit risk

Econometric scoring (2)

logit or probit regression

alternative to Z-scores of E. Altman

$$\underbrace{Y_t}_{[0,1]} \begin{pmatrix} 0 \\ 1 \\ \vdots \end{pmatrix} = fn\left[\beta_1 \underbrace{X_{1\,t-1}}_{D/E} \begin{pmatrix} 2 \\ 10 \\ \vdots \end{pmatrix} + \beta_2 \underbrace{X_{2\,t-1}}_{ICR} \begin{pmatrix} 7 \\ 3 \\ \vdots \end{pmatrix} + \beta_3 \underbrace{X_{3\,t-1}}_{ROA} \begin{pmatrix} 20\% \\ 10\% \\ \vdots \end{pmatrix} + \beta_4 \underbrace{X_{4\,t-1}}_{\dots} \begin{pmatrix} \dots \\ \dots \\ \vdots \end{pmatrix} \right]$$

$$0.05 = fn\left(+ \hat{\beta}_1 \, 4 + \hat{\beta}_2 \, 4 + \hat{\beta}_3 \, 13\% + \hat{\beta}_4 \dots \right)$$

Modeling credit spreads (3)

Strcutural Models – BOTTOM-UP approach

Credit spreads

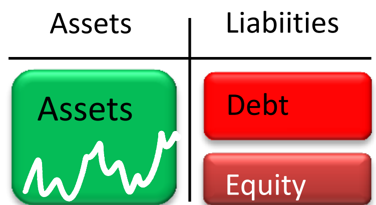
↑

PD, LGD

↑

Modeling the value of shareholders and debtholders depending on the capital structure and against the asset value

↑



MC simulations
binomial trees
Merton's option pricing model

Reduced-form Models – TOP-DOWN

(I believe the efficiency of markets & the price discovery process)

Credit spreads

↓

PD, LGD

The reduced-form approach(es)

- A starting point

$$D_0^{rf} = F\, e^{-rf \times T}$$

$$D_0^{risky} = F\, e^{-y \times T}$$

hyp: 0-coupon bond!

- The credit spread being

$$cs = y - rf$$

$$y = -\frac{1}{T}\ln\left(\frac{D_0^{risky}}{F}\right)$$

- The FX analogy (Jarrow & Turnbull)

$$\frac{D_0^{risky}}{D_0^{rf}} = e^{-(y-rf)T} = e^{-csT} = \varepsilon_T$$

- If default is a possibility...

$$E\left[D_T\right] = F\left(1 - P_{def}\right) + P_{def}\,\mathrm{E}\left[R \mid default\right]$$

$$= F\left(1 - P_{def}\right) + P_{def}\left(F - \mathrm{E}\left[Loss \mid default\right]\right)$$

$$= F - P_{def}\, LGD$$

EL

The reduced-form approach(es) (2)

- Therefore…

$$D_0^{rf} = F\,e^{-rf\times T}$$

$$D_0^{risky} = F\,e^{-y\times T}$$

$$= \left(F - P_{def}^{rn}\,LGD^{rn}\right)e^{-rf\times T}$$

- Or…

$$D_0^{risky} = \left(F - P_{def}^{h}\,LGD^{h}\right)e^{-(rf+crp)\times T}$$

- Which means…

$$y = rf + \underbrace{hel + crp}_{cs}$$

risk-neutral measure

risk-free expectation

still risky.

historical expected loss

risk premium as an opportunity cost.

$E[R_{stock}] = rf + \beta.(E(R_m) - rf) + \varepsilon$

Example

A B-rated bond is trading at €94 and matures in one year from now. The risk-free rate is 4% per annum. Historical and risk-neutral LGDs (in %) are estimated at 40%.
The yield of this bond is:

$$y = -\frac{1}{T}\ln\left(\frac{D_0^{risky}}{F}\right) = -\ln\left(\frac{94}{100}\right) = 6.188\%,$$

or a credit spread of : $cs = 6.188\% - 4\% = 2.188\%$

Looking at Table 1.1, we can find that the historical one-year probability for B-rated counterparts is 4.047%. Using this, we can deduce how much is the implied credit risk premium (crp) to obtain the same market price from the risky expected payoff of the bond:

$$= \left(F - P_D^h\, LGD^h\right) e^{-(r_f + crp)T}$$

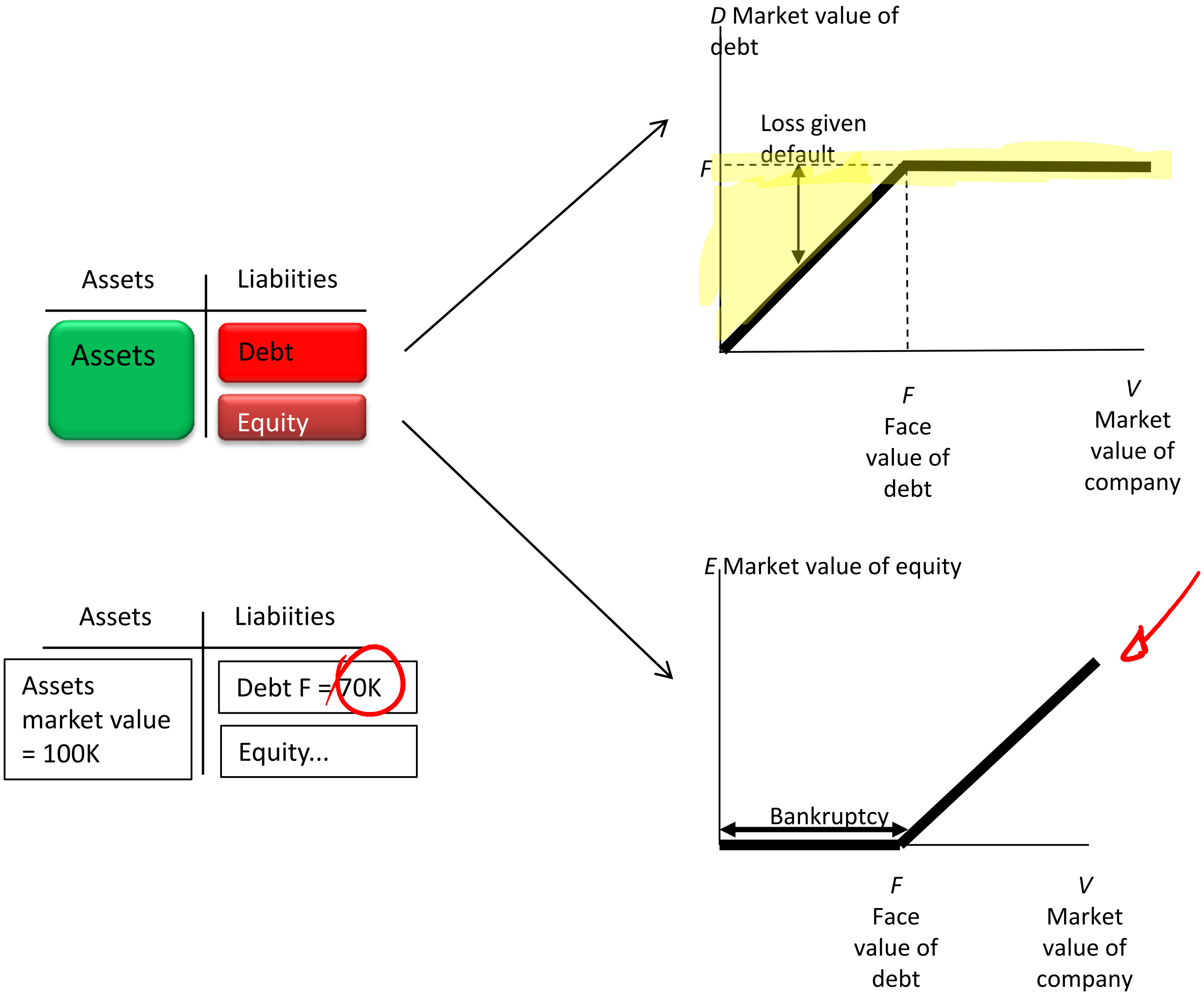
$$\rightarrow crp = -\frac{1}{T}\ln\left(\frac{D_0^{risky}}{\left(F - P_D^h\, LGD^h\right)}\right) - r_f = -\ln\left(\frac{94\%}{100 - 4.047\% \times 40}\right) - 4\%$$

$$= 0.555\%$$

Thefore, the credit spread can be decomposed as follows:

$$cs = hel + crp = 1.632\% + 0.555\% = 2.188\%.$$

The structural approach (Merton) – step 1



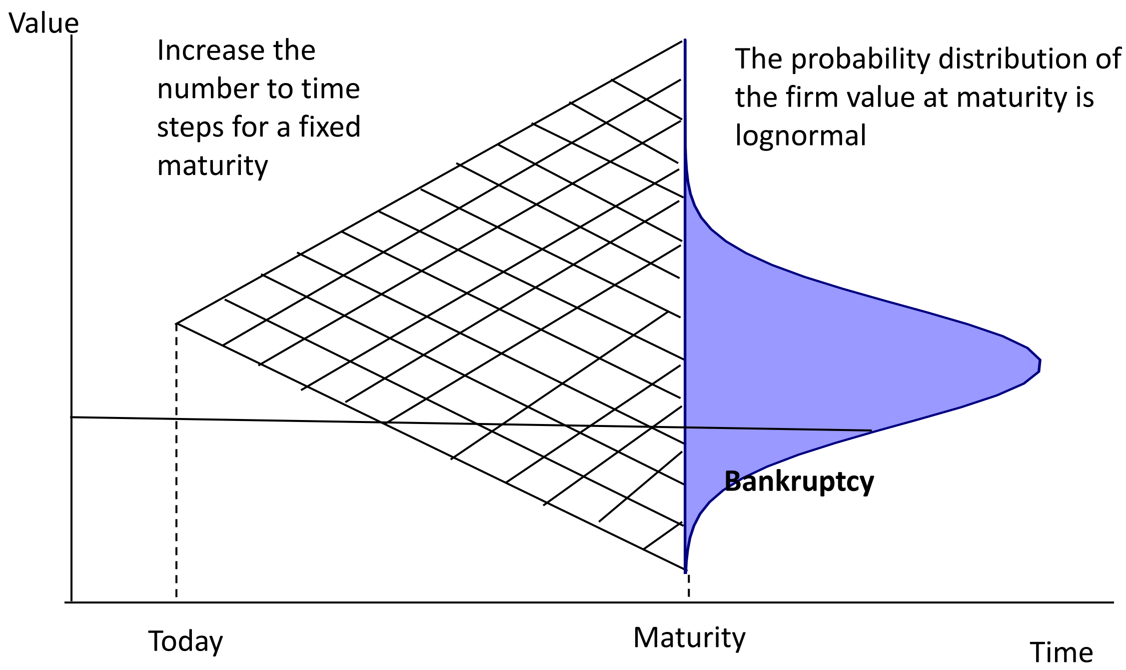
$D_T = \min(V_A, F)$

$= F - \max(F - V_A, 0)$

	$V_A < F$	$V_A > F$
D	V_A	70
E	0	$V_A - 70$

Now, we know that…

- Options can be valued in two ways
  - » Binomial model
  - » Continuous-time model: Black-Scholes(-Merton) formula



A basic example

Assets	Liabiities
Assets market value = 100K	Debt F = 70K
	Equity...

- Other parameters
  - » Volatility of asset variations: 40%
  - » Risk-free rate: 5%
  - » Maturity of debt: 1 year

Structural models (Merton’s idea)
> Using the binomial pricing technique

Merton Model: example using binomial pricing

Data:
Market Value of Unlevered Firm: 100,000
Risk-free rate per period: 5%
Volatility: 40%

Company issues 1-year zero-coupon
Face value = 70,000
Proceeds used to pay dividend or to buy back shares

Binomial option pricing: review

Up and down factors: $u = e^{\sigma\sqrt{\Delta t}} = 1.492$ $\quad d = \frac{1}{u} = .670$

Risk neutral probability :

$$p = \frac{1 + r_f - d}{u - d} = \frac{1.05 - .67}{1.492 - 0.670} = .462$$

1-period valuation formula

$$f = \frac{p f_u + (1-p) f_d}{1 + r_f}$$

$$E = \frac{0.462 \times 79{,}182 + 0.538 \times 0}{1.05}$$

$$D = \frac{0.462 \times 70{,}000 + 0.538 \times 67{,}032}{1.05}$$

Today:
V = 100,000
E = 34,854
D = 65,146

Up state:
V = 149,182
E = 79,182
D = 70,000

Down state:
V = 67,032
E = 0
D = 67,032

$\Delta t = 1$

Calculating the cost of borrowing

- Spread = Borrowing rate – Risk-free rate
  - » Borrowing rate = Yield to maturity on risky debt
  - » For a zero coupon (using annual compounding): $D = \frac{F}{(1+y)^T}$

- In our example: $65{,}146 = \frac{70{,}000}{1+y}$

y = 7.45%

Spread = 7.45% - 5% = 2.45% (245 basis points)

Decomposing the value of the risky debt

In our simplified model:

$$D = \frac{F}{1+r_f} \times p + \frac{V_d}{1+r_f} \times (1-p)$$

$$D = \frac{F}{1+r_f} - \frac{(1-p)(F-V_d)}{1+r_f}$$

$$D = \frac{70{,}000}{1.05} \times 0.462 + \frac{67{,}032}{1.05} \times .538$$

$$= 66{,}667 \times 0.462 + 63{,}840 \times .538$$

$$= 65{,}146$$

LGD PD

$$D = \frac{70{,}000}{1.05} - \frac{70{,}000 - 67{,}032}{1.05} \times .538$$

$$= 66{,}667 - 2{,}827 \times .538$$

$$= 65{,}146$$

F: loss given default if no recovery

V_d : recovery if default

$F - V_d$: loss given default

$(1 - p)$: risk-neutral probability of default

Pdef = = $N(-d_2)$ in B&Scholes

Weighted Average Cost of Capital

1. Start from WACC for unlevered company
   - As V does not change, $WACC$ is unchanged
   - Assume that the CAPM holds

$$WACC = k_A = k_f + (R_M - r_f)\beta_A$$

   - Suppose: $\beta_A = 1$ $R_M - r_f = 6\%$

$$WACC = 5\% + 6\% \times 1 = 11\%$$

2. Use $WACC$ formula for levered company to find rE

$$k_A = k_E \frac{E}{V} + k_D \frac{D}{V} \qquad 11\% = k_E \frac{34,854}{100,000} + k_D \frac{65,146}{100,000}$$

$$\beta_A = \beta_E \frac{E}{V} + \beta_D \frac{D}{V} \qquad 1 = \beta_E \frac{34,854}{100,000} + \beta_D \frac{65,146}{100,000}$$

Cost (beta) of equity

- Remember : $C = Delta_{call} \times S - B$
  - A call can be seen as a portfolio of the underlying asset combined with borrowing B.
- The fraction invested in the underlying asset is
  - $X = (Delta_{call} \times S) / C$
- The beta of this portfolio is $X\, \beta_{asset}$
- When analyzing a levered company:
  - call option = equity
  - underlying asset = value of company
  - $X = V/E = (1+D/E)$

$$\beta_E = \beta_A \times Delta \times \frac{V}{E} = \beta_A \times Delta \times \left(1 + \frac{D}{E}\right)$$

Reminder:

$$Delta = \frac{f_u - f_d}{uS - dS}$$

In example:

$\beta_A = 1$

$Delta_E = 0.96$

$V/E = 2.87$

$\beta_E = 2.77$

$k_E = 5\% + 6\% \times 2.77$

$= 21.59\%$

Cost (beta) of debt

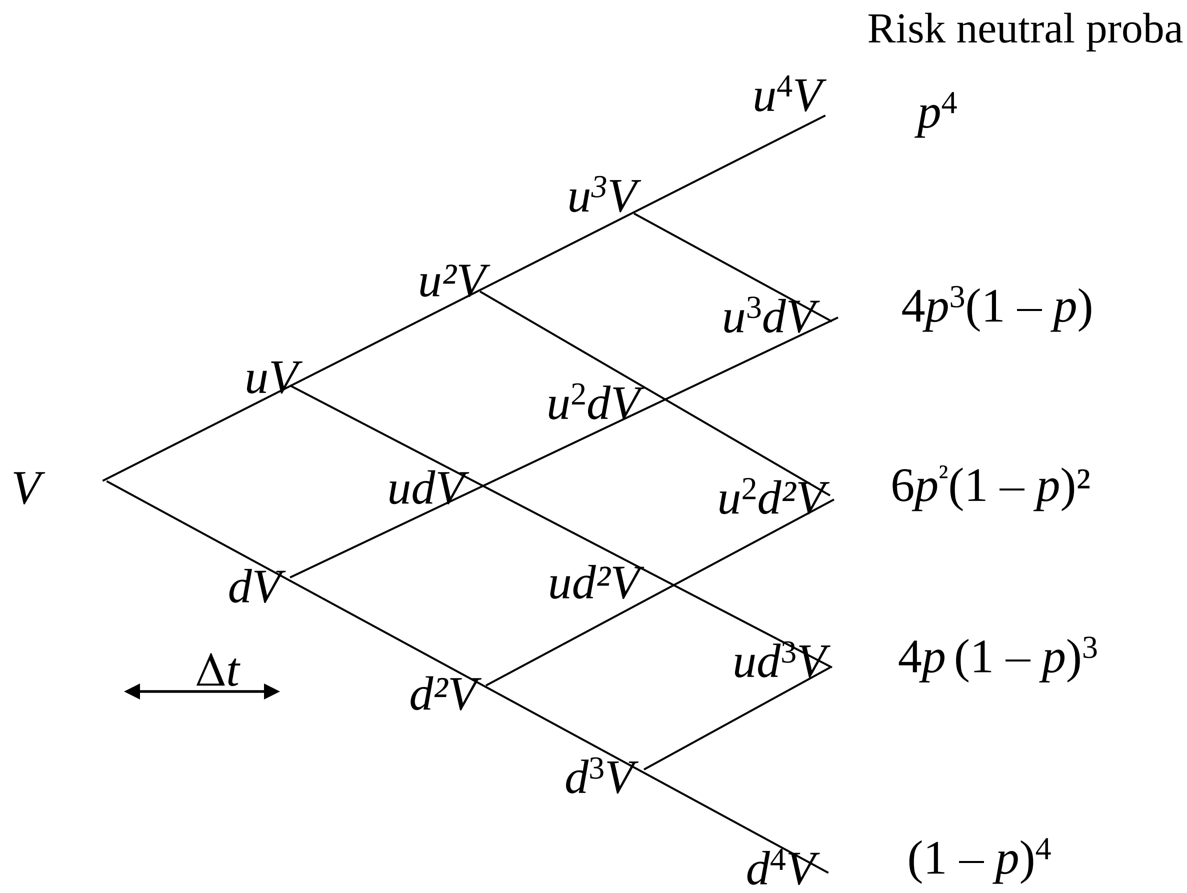
- Remember : $D = PV(FaceValue) - Put$

$$Delta_D = \frac{(F - Put_u) - (F - Put_d)}{uS - dS} = -\frac{Put_u - Put_d}{uS - dS} = -Delta_{Put}$$

- $Put = Delta_{put} \times V + B$ (!! $Delta_{put}$ is negative: $Delta_{put} = Delta_{call} - 1$)
  - » So : $D = PV(FaceValue) - Delta_{put} \times V - B$
  - » Fraction invested in underlying asset is $X = - Delta_{put} \times V/D$
  - » $\beta_D = - \beta_A \, Delta_{put} \, V/D$

In example:
$\beta_A = 1$
$Delta_D = 0.04$
$V/D = 1.54$
$\beta_D = 0.06$
$k_D = 5\% + 6\% \times 0.09$
$= 5.33\%$

Multiperiod binomial valuation



For European option,
(1) At maturity, calculate
- firm values;
- equity and debt values
- risk neutral probabilities

(2) Calculate the expected values in a neutral world
(3) Discount at the risk free rate

Multiperiod binomial valuation: example

Firm issues a 2-year zero-coupon
Face value = 70,000
V = 100,000
Int.Rate = 5% (annually compounded)
Volatility = 40%
Beta Asset = 1

4-step binomial tree Δt = 0.50
u = 1.327, d = 0.754
r_f = 2.47% per period = $(1.05)^{1/2}$-1
p = 0.473

					# paths	Proba/path	Proba	E	D
				309,990	1	0.050	0.050	239,990	70,000
			233,621						
		176,065		176,065	4	0.056	0.223	106,065	70,000
	132,690		132,690						
100,000		100,000		100,000	6	0.062	0.373	30,000	70,000
	75,364		75,364						
		56,797		56,797	4	0.069	0.277	0	56,797
			42,804						
				32,259	1	0.077	0.077	0	32,259
						Expected values		46,823	63,427
						Present values		42,470	57,530

Multiperiod valuation: details

Firm value

Down					
0	100,000	132,690	176,065	233,621	309,990
1		75,364	100,000	132,690	176,065
2			56,797	75,364	100,000
3				42,804	56,797
4					32,259

Σ = 100

Equity value

42,470	69,427	109,399	165,308	239,990
	20,280	36,828	64,377	106,065
		6,388	13,843	30,000
			0	0
				0

Delta

0.86	0.95	1.00	1.00
	0.70	0.88	1.00
		0.43	0.69
			0.00

Beta

2.02	1.82	1.61	1.41
	2.62	2.39	2.06
		3.78	3.78
			#DIV/0!

Debt value

57,530	63,262	66,667	68,313	70,000
	55,084	63,172	68,313	70,000
		50,409	61,521	70,000
			42,804	56,797
				32,259

Delta

0.14	0.05	0.00	0.00
	0.30	0.12	0.00
		0.57	0.31
			1.00

Beta

0.25	0.10	0.00	0.00
	0.40	0.19	0.00
		0.65	0.37
			1.00

Multiperiod binomial valuation: additional details

- From the previous calculation, we can decompose *D* into:
  - ✓ Risk-free debt
  - ✓ Risk-neutral probability of default
  - ✓ Expected loss given default
- Expected value at maturity:
  - ✓ Risk-free debt = 70,000
  - ✓ Default probability = 0.354
  - ✓ Expected loss given default = 18,552
  - ✓ Risky debt = 70,000 – 0.354 × 18,552 = 63,427
- Present value:
  - ✓ $D = 63{,}427 / (1.05)^2 = 57{,}530$

Structural models (Merton's idea)
> Using the Black & Scholes option pricing model
(continuous modelling)

Continuous model (reminder)

- From the real options course, we know that...
  - Value at maturity of a call, e.g.

$$C_T = (S_T - K)^+ = \max(S_T - K, 0)$$

  - Thus, the value at t=0

$$C_0 = \mathrm{E}\left[e^{-rT} (S_T - K)^+ \right]$$

$$= e^{-rT} \mathrm{E}\left[(S_T - K) 1_{\{S_T > K\}} \right]$$

$$= e^{-rT} \mathrm{E}\left[(S_T - K) 1_{\{S_T > K\}} \right] = e^{-rT} \mathrm{E}\left[S_T \, 1_{\{S_T > K\}} \right] - e^{-rT} K \, \mathrm{E}\left[1_{\{S_T > K\}} \right]$$

$$= S_0 N(d_1) - e^{-rT} K \, N(d_2)$$

✓ The valuation difficulty is of course in the last step and was first demonstrated with the PDE approach and then with the equivalent martingale measure approach.

The (Merton) structural model (2)

- Debt can be seen as...

$$D_T = \min\left(F, V_T\right)$$

$$= F - \max\left(F - V_T, 0\right)$$

$$D_0 = Fe^{-rfT} - Put$$

$$= Fe^{-rfT} - \left[-V_0\ N\left(-d_1\right) + Fe^{-rfT}\ N\left(-d_2\right)\right]$$

$$= V_0\ N\left(-d_1\right) + Fe^{-rfT}\ N\left(d_2\right)$$

$$= Fe^{-rfT} - N\left(-d_2\right)\left[Fe^{-rfT} - V_0 \frac{N\left(-d_1\right)}{N\left(-d_2\right)}\right]$$

N(-d₂) = PD

PV(Recovery).

PD

PV[LGD]

$$cs = -\frac{1}{T}\ln\left[\frac{V_0}{Fe^{-rfT}}\ N\left(-d_1\right) + N\left(d_2\right)\right]$$

Merton Model: example

Data	
Market value unlevered firm	€100,000
Risk-free interest rate (an.comp):	5%
Beta asset	1
Market risk premium	6%
Volatility unlevered	40%

Company issues 2-year zero-coupon
Face value = €70,000
Proceed used to buy back shares

Using Black-Scholes formula	
Price of underling asset	100,000
Exercise price	70,000
Volatility σ	0.40
Years to maturity	2
Interest rate	5%
Value of call option	41,772
Value of put option (using put-call parity)	
C+PV(ExPrice)-Sprice	5,264

Details of calculation:

PV(ExPrice) = 70,000/(1.05)^2= 63,492

log[Price/PV(ExPrice)] = log(100,000/63,492) = 0.4543

$\sigma\sqrt{t}$ = 0.40 $\sqrt{2}$ = 0.5657

$d1$ = log[Price/PV(ExPrice)]/ $\sigma\sqrt{}$ + 0.5 $\sigma\sqrt{}$ t = 1.086

$d2 = d1 - \sigma\sqrt{}\ t$ = 1.086 - 0.5657 = 0.520

$N(d1)$ = 0.861

$N(d2)$ = 0.699

C = $N(d1)$ *Price* - $N(d2)$ *PV*(*ExPrice*)
= 0.861 × 100,000 - 0.699 × 63,492
= 41,772

Valuing the risky debt

- Market value of risky debt = Risk-free debt – Put Option

$$D = e^{-rT} F - \{- V[1 - N(d_1)] + e^{-rT} F [1 - N(d_2)]\}$$

- Rearrange:

$$D = e^{-rT} F N(d_2) + V [1 - N(d_1)]$$

$$D = e^{-rT} F \times N(d_2) + V \frac{1 - N(d_1)}{1 - N(d_2)} \times [1 - N(d_2)]$$

Value of risk-free debt	×	Probability of no default	+	Discounted expected recovery given default	×	Probability of default

Example (continued)

$$D = V - E = 100{,}000 - 41{,}772 = 58{,}228$$

$$D = e^{-rT} F - \text{Put} = 63{,}492 - 5{,}264 = 58{,}228$$

$$D = e^{-rT} F \times N(d_2) \quad + \quad V \frac{1-N(d_1)}{1-N(d_2)} \times [1-N(d_2)]$$

$$= 63{,}492 \times 0.6985 + 46{,}031 \times 0.3015 = 58{,}228$$

$$V \frac{1-N(d_1)}{1-N(d_2)} = 100{,}000 \frac{1-0.8612}{1-0.6985} = 46{,}031$$

Expected amount of recovery

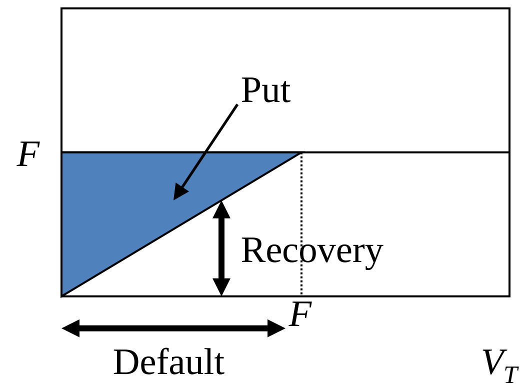
- We want to prove: $E[V_T / V_T < F] = V\,e^{rT}[1 - N(d_1)]/[1 - N(d_2)]$
  - » Recovery if default = V_T
  - » Expected recovery given default = $E[V_T / V_T < F]$

  (mean of truncated lognormal distribution)

- The value of the put option:
  - » $P = -V\,N(-d_1) + e^{-rT} F\,N(-d_2)$

- can be written as
  - » $P = e^{-rT} N(-d_2)[- V\,e^{rT} N(-d_1)/N(-d_2) + F]$

Discount factor	Probability of default	Expected value of put given



- But, given default: $V_T = F - Put$
- So: $E[V_T / V_T < F] = F - [- V\,e^{rT} N(-d_1)/N(-d_2) + F] = V\,e^{rT} N(-d_1)/N(-d_2)$

Another presentation

$$D = e^{-rT}\left\{F - [1-N(d_2)] \times [F - Ve^{rT}\frac{1-N(d_1)}{1-N(d_2)}]\right\}$$

Discount factor

Face Value

Probability of default

Loss if no recovery

Expected Amount of recovery given default

Expected loss given default

$$D = 0.9070\{100{,}000 - 0.3015 \times [70{,}000 - 50{,}749]\}$$

Example using Black-Scholes

Data

Market value unlevered company	€ 100,000
Debt = 2-year zero coupon Face value	€ 60,000
Risk-free interest rate	5%
Volatility unlevered company	30%

Using Black-Scholes formula

Market value unlevered company	€ 100,000
Market value of equity	€ 46,626
Market value of debt	€ 53,374
Discount factor	0.9070
$N(d_1)$	0.9501
$N(d_2)$	0.8891

Using Black-Scholes formula

Value of risk-free debt € 60,000 x 0.9070 = 54,422

Probability of default
$N(-d_2) = 1\text{-}N(d_2) = 0.1109$

Expected recovery given default
$V e^{rT} N(-d_1)/N(-d_2)$
= (100,000 / 0.9070) (0.05/0.11)
= 49,585

Expected recovery rate | default
= 49,585 / 60,000 = 82.64%

Calculating borrowing cost

Initial situation

Balance sheet (market value)

Assets	100,000	Equity	100,000

Note: in this model, market value of company doesn't change (Modigliani Miller 1958)

Final situation after:

issue of zero-coupon & shares buy back

Balance sheet (market value)

Assets	100,000	Equity	41,772
		Debt	58,228

Yield to maturity on debt y:

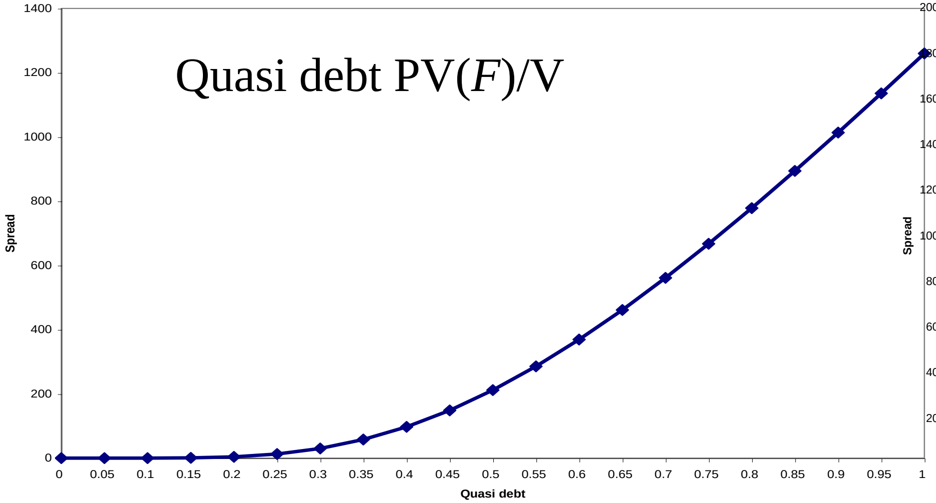
$D = FaceValue/(1+y)^2$

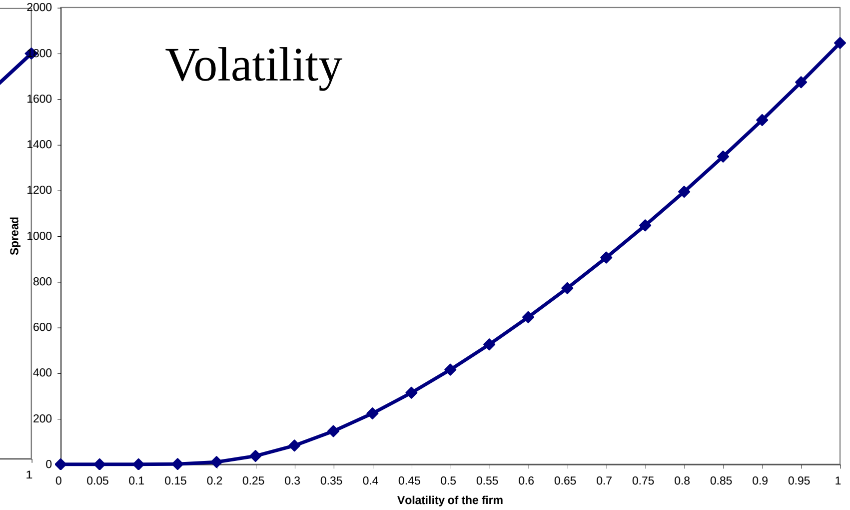
$58{,}228 = 60{,}000/(1+y)^2$

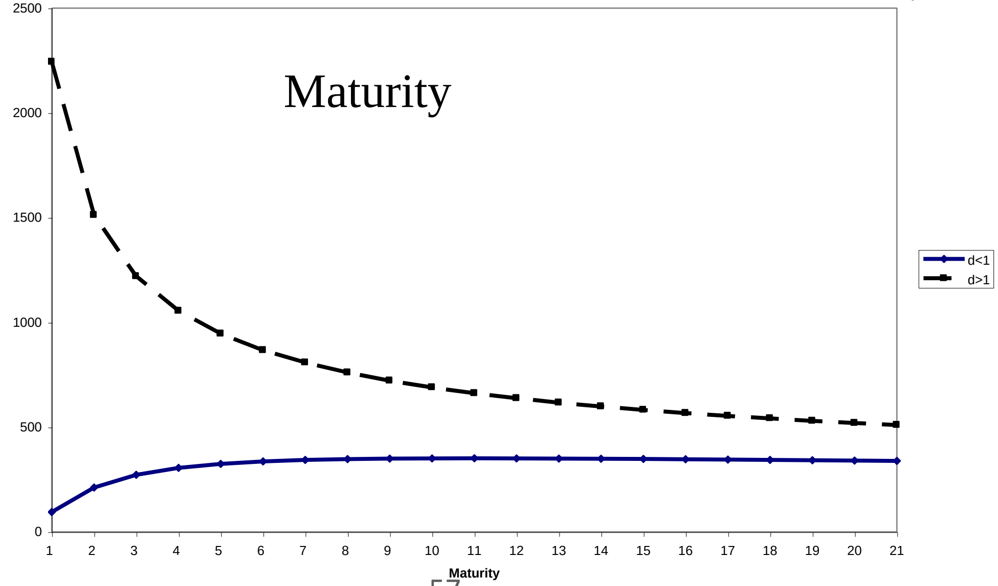
$y = 9.64\%$

Spread = 364 basis points (bp)

Determinant of the spreads



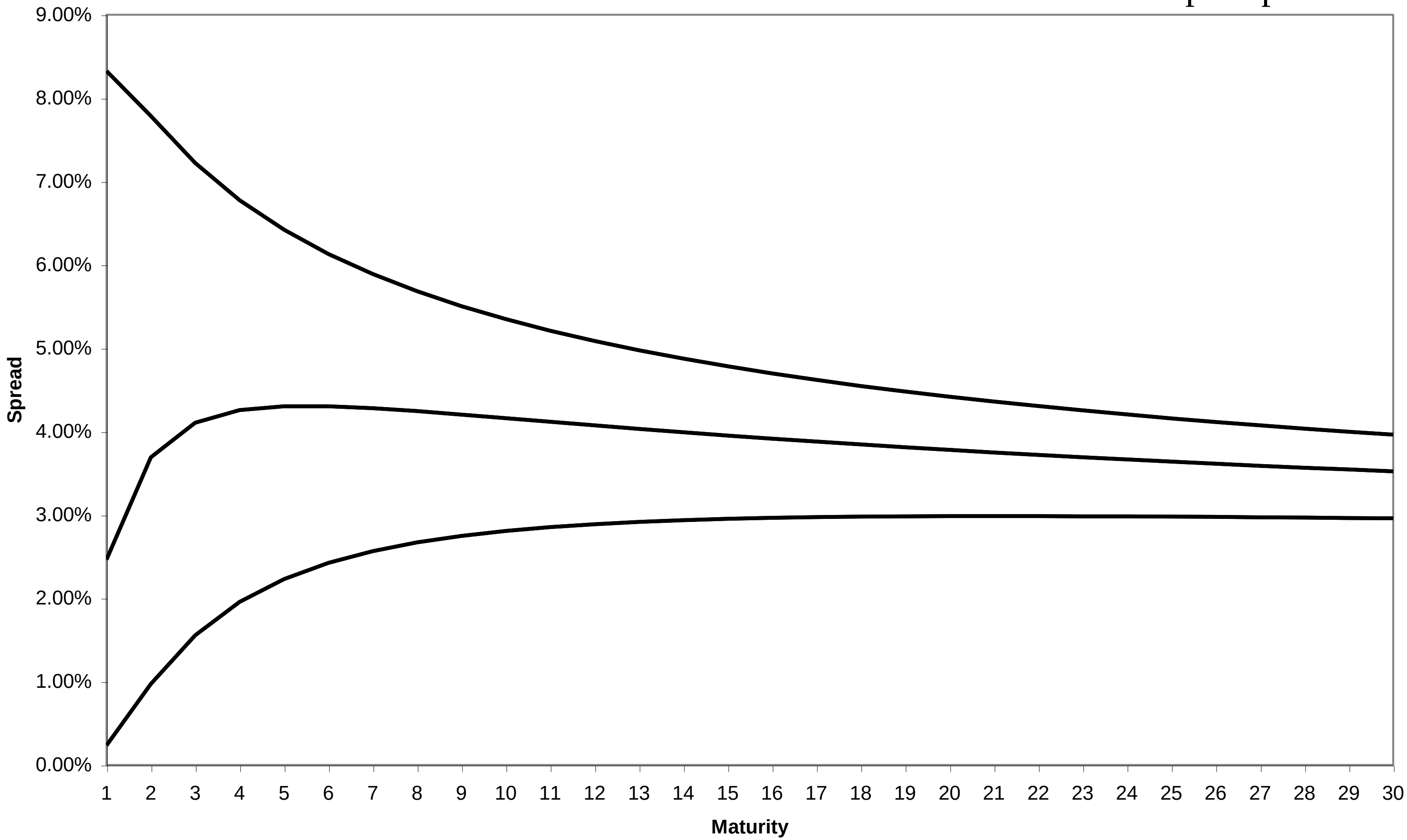




Maturity and spread

$$s = -\frac{1}{T}\ln(N(d_2) + \frac{1}{d}N(-d_1))$$

Proba of no default - Delta of put option



Inside the relationship between spread and maturity

Spread (σ = 40%)

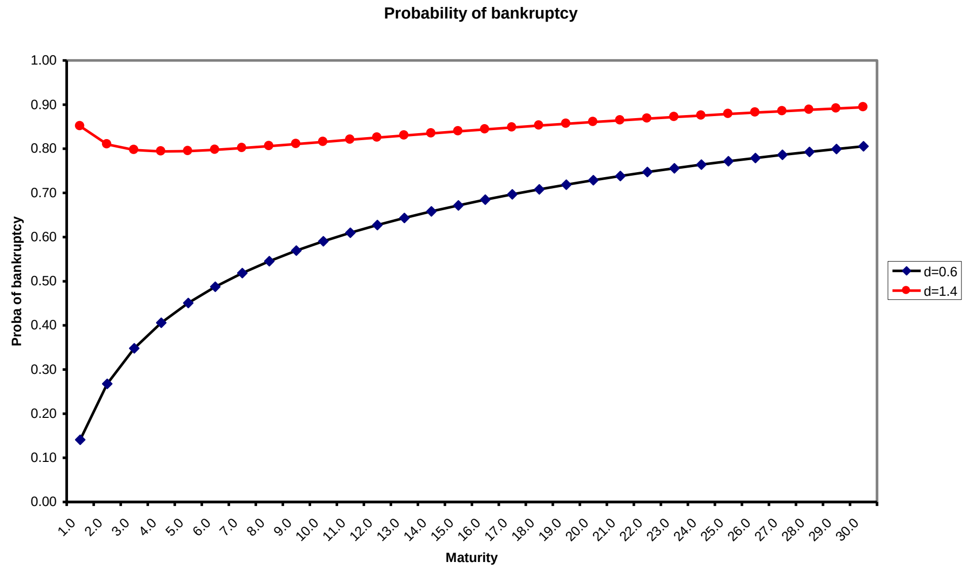
	d = 0.6	d = 1.4
T = 1	2.46%	39.01%
T = 10	4.16%	8.22%

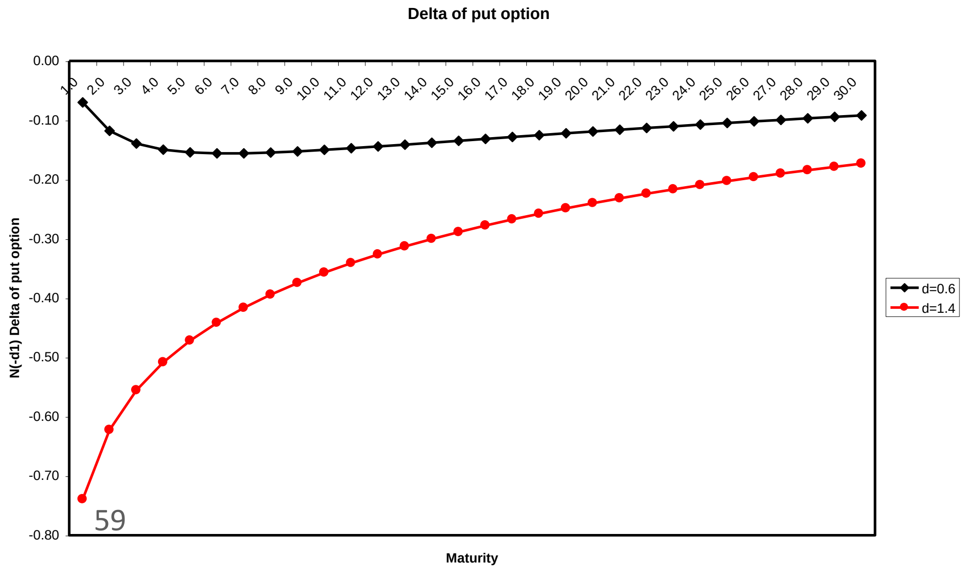
Probability of bankruptcy

	d = 0.6	d = 1.4
T = 1	0.14	0.85
T = 10	0.59	0.82

Delta of put option

	d = 0.6	d = 1.4
T = 1	-0.07	-0.74
T = 10	-0.15	-0.37





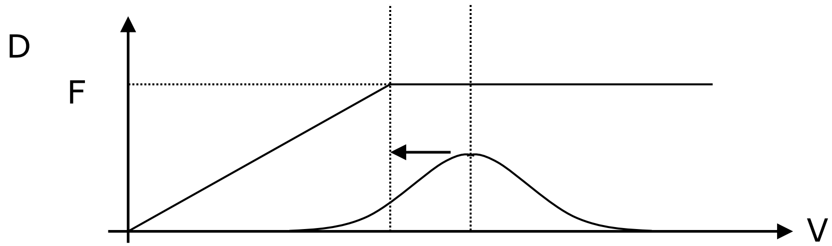
Structural models (Merton’s idea) > Beyond Merton’s straightforward model

Merton: ...but

- Restrictive hypothesis
  - 0-coupon bond
  - Constant interest rate
  - A single bond issue
  - « Perfect markets »
- Nice principle but poor pricing performance
- Thus:
  - Use it to put a qualitative rating and to explain incentives , determinants and use it as a scorecard...
  - But do not expect « 1bp » pricing match!
- Implementation: what do you need?

Merton: Keeping the general idea

- The option principle applied to a « distance-to-default » = structural model



$N(d_{1,2})$

$$\frac{\ln\left[V / Fe^{-rT}\right]}{\sigma_V \sqrt{T}} \approx \frac{V-F}{\sigma_V} \approx \frac{\text{distance}}{\text{in units of volatility}} \approx \frac{\text{distance}}{\text{speed}} \approx \text{scaled distance-to-default}$$

(DD)

- Firm-specific components
  - When default risk ↗, E→0, D→recovery rate
  - default risk = f(economy,firm-specific components)
- KMV application of Merton: Mapping to ratings following empirical evidence
  - Follow evolution of default risk in continuous time → Continuous-time evolution of creditworthiness

KMV's procedure: Introduction

- Basis:
  - » Straight application of Merton with
    - ✓ Some extensions in terms of « smiles », etc...
    - ✓ A scaling idea of EDF against rating ranks, thanks to the computation of « distance-to-default » values.
- Moody's KMV Expected Default Frequency (EDF™) credit risk measures :
  - » forward-looking default probabilities
  - » for public and private companies
  - » actual probabilities of default
  - » built from over 15 years of experience with market and fundamental data and modeling
  - » Public company EDF credit measures are based on extracting collective, real-time intelligence from markets globally. A public firm's probability of default is calculated from three drivers—the market value of its assets, its volatility, and its current capital structure. For each firm, the EDF credit measure captures the distilled credit insight from the equity market and combines it with a detailed picture of the company's current capital structure. »

KMV’s procedure: Introduction (2)

- Private company EDF measures :
  - ✓ Using Moody’s KMV proprietary Credit Research Database™ (CRD). Fundamental data on private firms are lined up with extensive observations of default to capture the predictors and their impact on default.
  - ✓ Private company credit risk drivers differ across countries

    → network of Moody’s KMV RiskCalc™ models that capture the fundamental drivers of default for private firms across a wide array of countries accounting for more than 75% of global GDP. »

KMV's procedure: Introduction (3)

E = call(V_A, σ_A, F, T, r..)

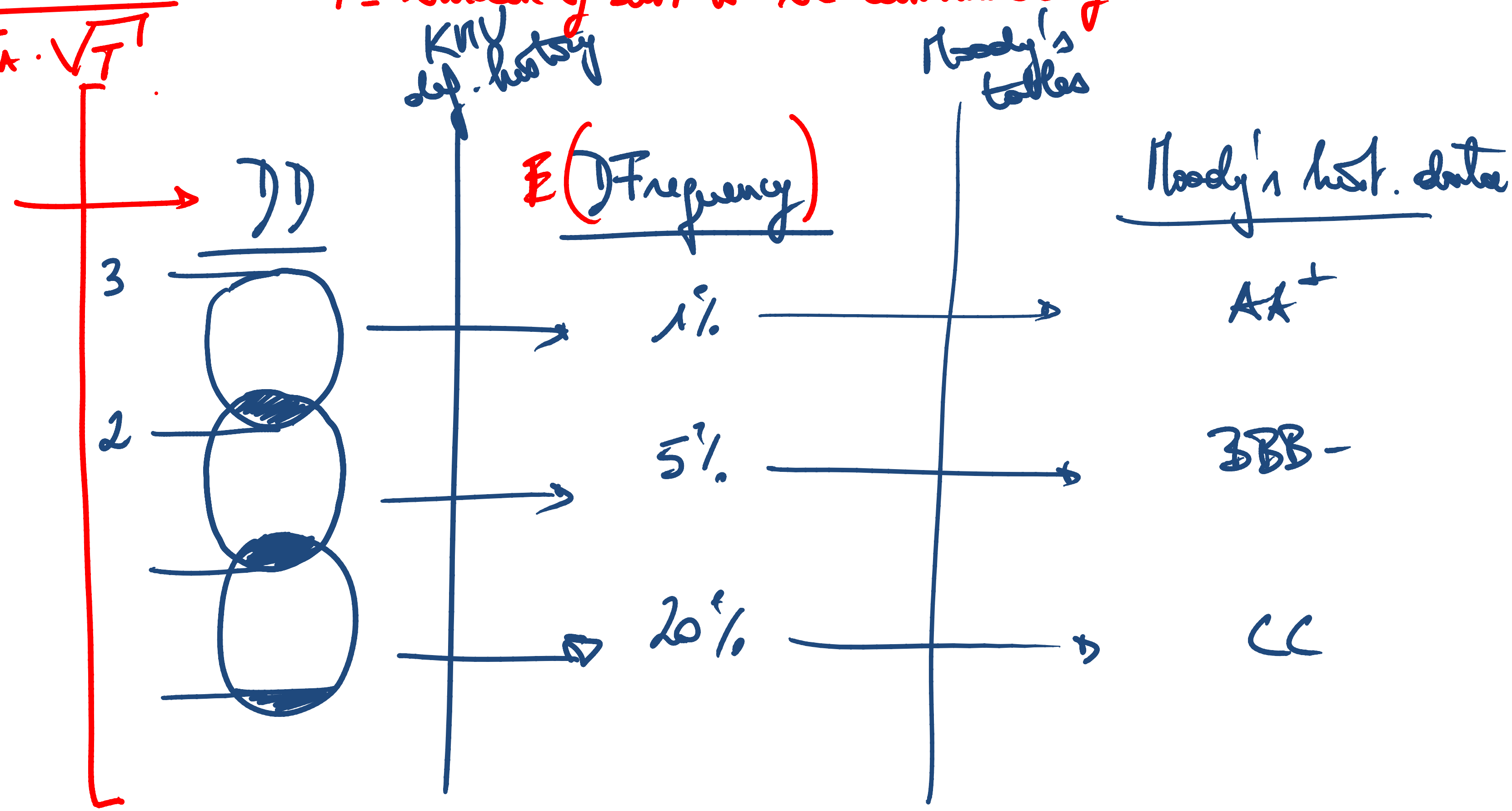
① huge database of default history (29 years)

$$\frac{\ln\left(V/Fe^{-rT}\right)}{\sigma_A \cdot \sqrt{T}}$$

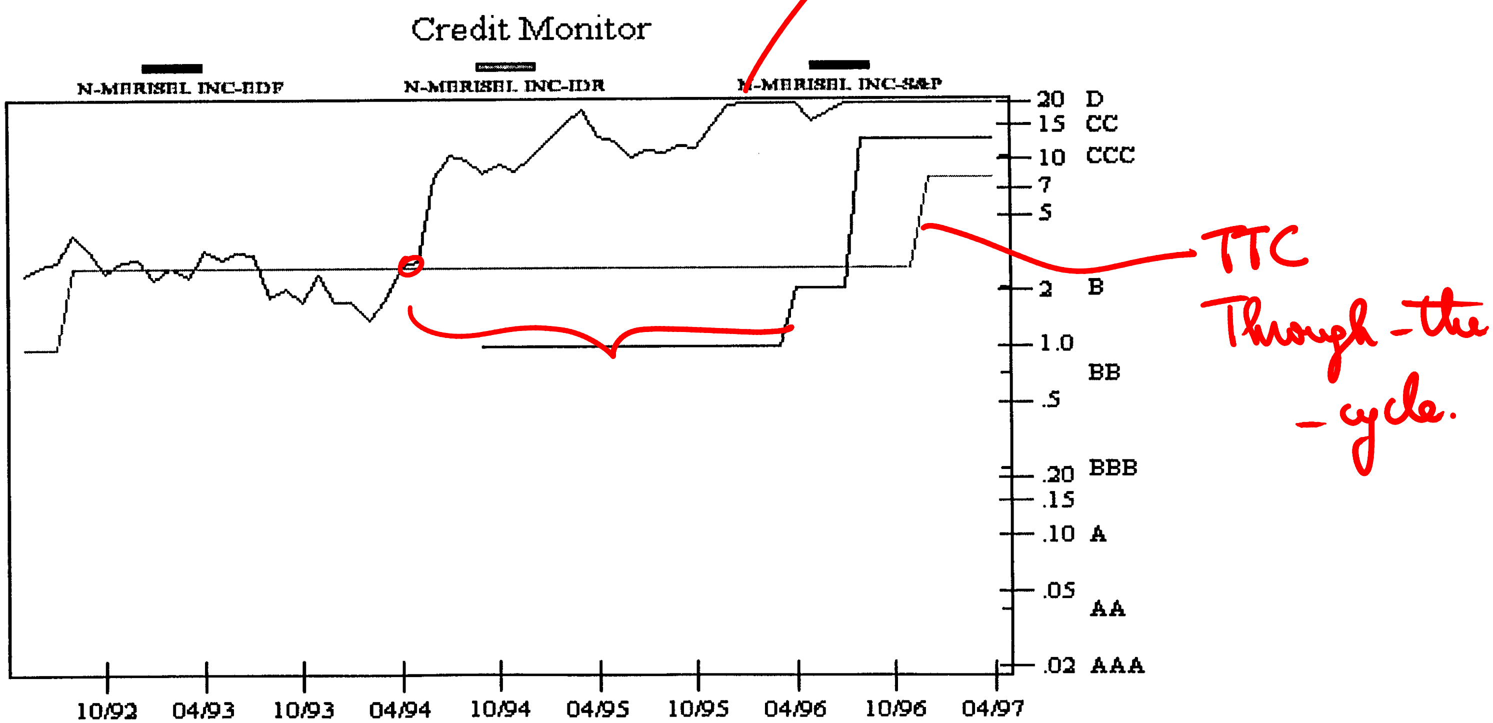
T = average maturity of debt

F = burden of debt to be reimbursed by T

σ_A, V_A

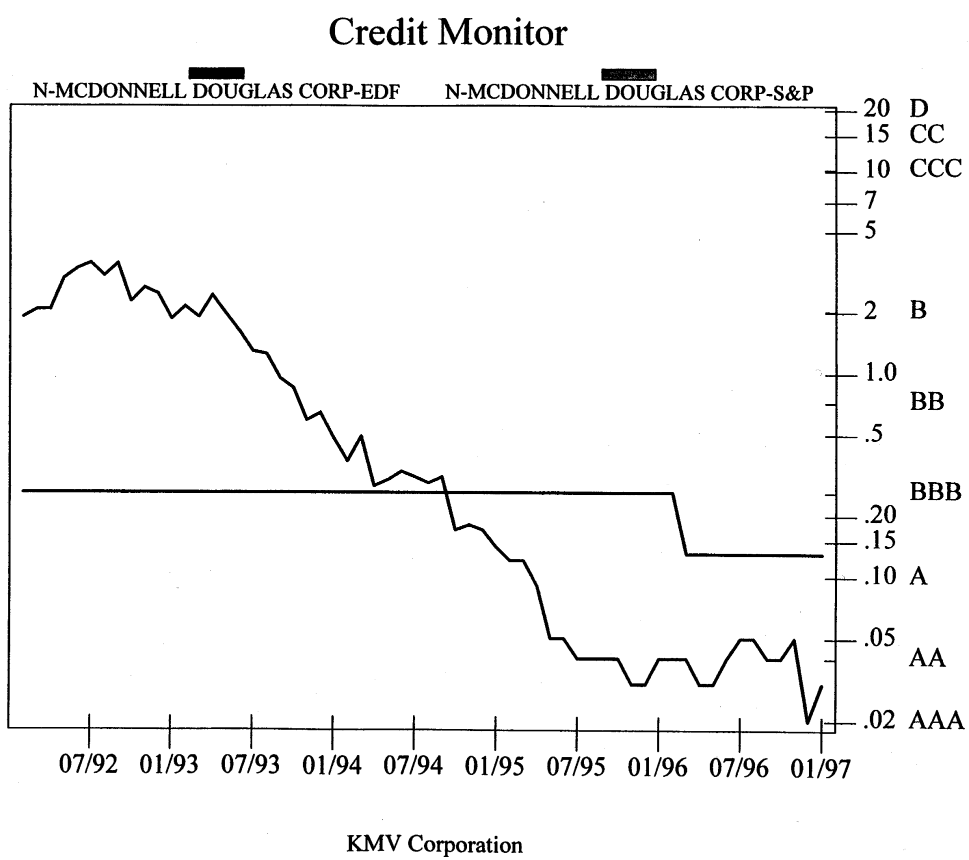


KMV's experience (1)



KMV Corporation

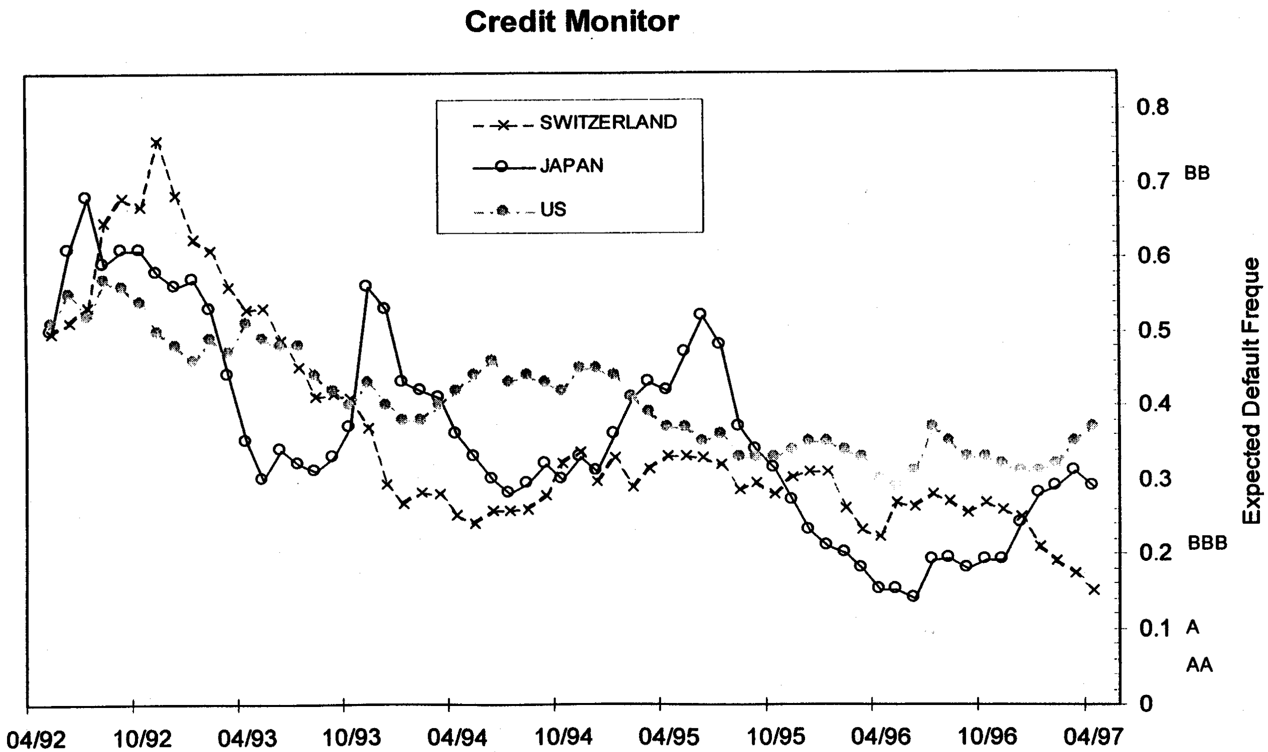
KMV's experience (2)



? how much is linked to micro vs macro phenomenon?

Studies still show that credit risk is more a systematic than specific risk

KMV's experience (3)



KMV Corporation

Merton: ...but

- Restrictive hypothesis
  - » 0-coupon bond
  - » Constant interest rate
  - » A single bond issue
  - » « Perfect markets »
- Nice principle but poor pricing performance
- Thus:
  - » Use it to put a qualitative rating and to explain incentives , determinants and use it as a scorecard...
  - » But do not expect « 1bp » pricing match!
- Implementation: what do you need?

Pirotte (1999)

- Credit spread behavior:

$$cs(T) = -\frac{1}{T}\ln\left[\begin{array}{l} N(d_2) + (B_0/V_0)^{2\gamma-2}N(l_2) \\ +\varphi\, q^F\ [N(-d_1) - N(-k_1) - (B_0/V_0)^{2\gamma}(N(h_1) - N(l_1))] \\ +\phi\,\frac{q^F}{q^H}\ [N(-k_2) + (B_0/V_0)^{2\gamma-2}N(h_2)] \end{array}\right]$$

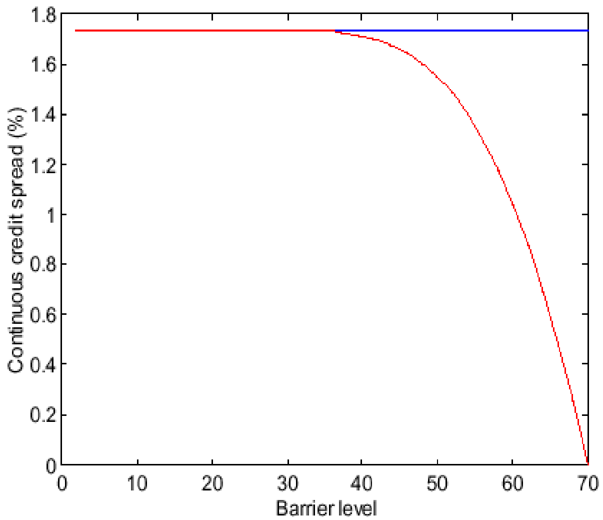


Figure 7. $V = 100$, $F = 70$, H between 2 and 70, $R = 5.43\%$, $\sqrt{T^-} = 30\%$, $\delta = 0\%$, $\varphi = 1$, $T = 5$. The straight line is Merton's credit spread.

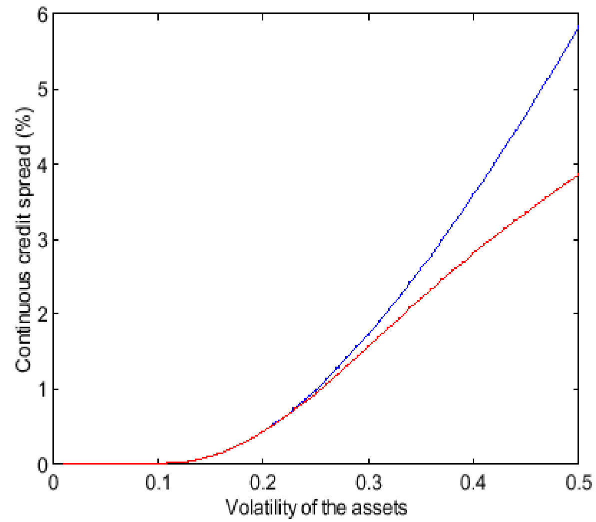


Figure 8. $V = 100$, $F = 70$, $H = 70\%$ of F, $R = 5.43\%$, $\sqrt{T^-}$ between 1% and 50%, $\delta = 0\%$, $\varphi = 1$, $T = 5$. The monotone increasing curve is Merton's credit spread.

Pirotte (1999)

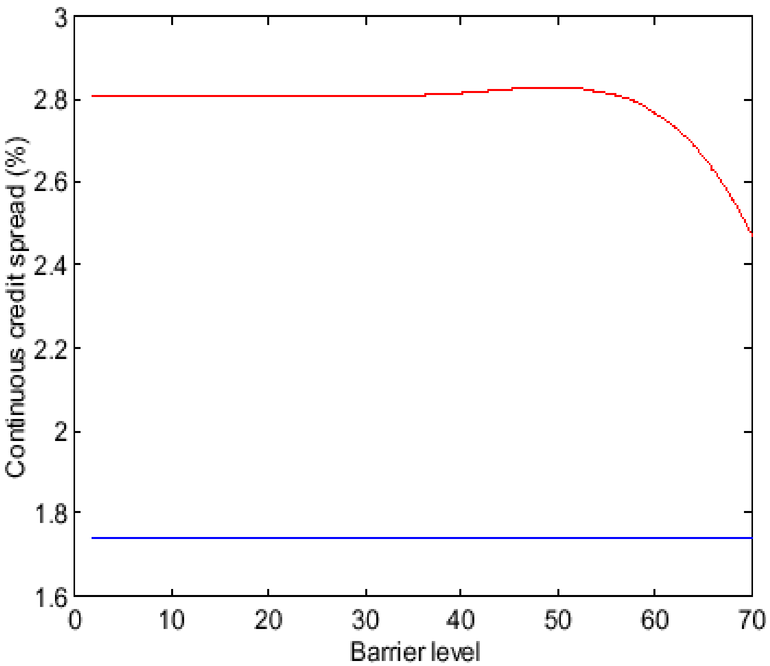


Figure 11. $V = 100$, $F = 70$, H between 2 and 70,

$R = 5.43\%$, $\sqrt{T^-} = 30\%$, $\delta = 0\%$, $\varphi = .75$, $T = 5$.

The straight line is Merton's credit spread.

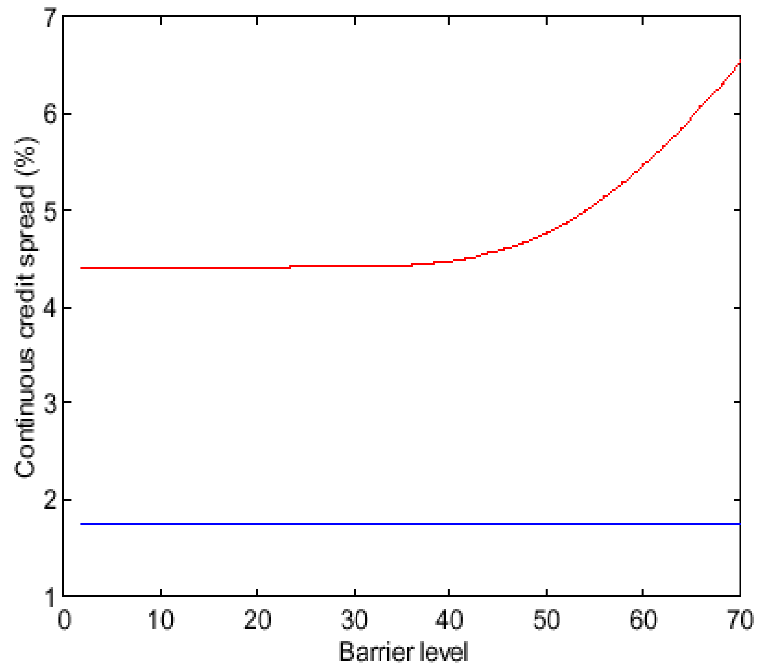


Figure 12. $V = 100$, $F = 70$, H between 2 and 70,

$R = 5.43\%$, $\sqrt{T^-} = 30\%$ $\delta = 0\%$, $\varphi = .4$, $T = 5$.

The straight line is Merton's credit spread.

Pirotte (1999)

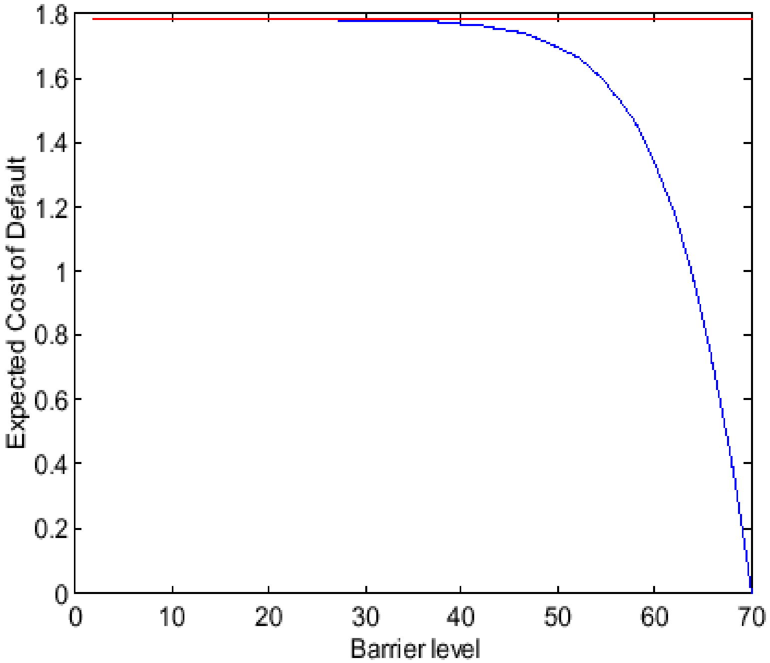


Figure 13. $V = 100$, $F = 70$, H between 2 and 70, $R = 5.43\%$, $\sqrt{T^{-}} = 20\%$, $\delta = 2\%$, $\varphi = 1$, $T = 5$.

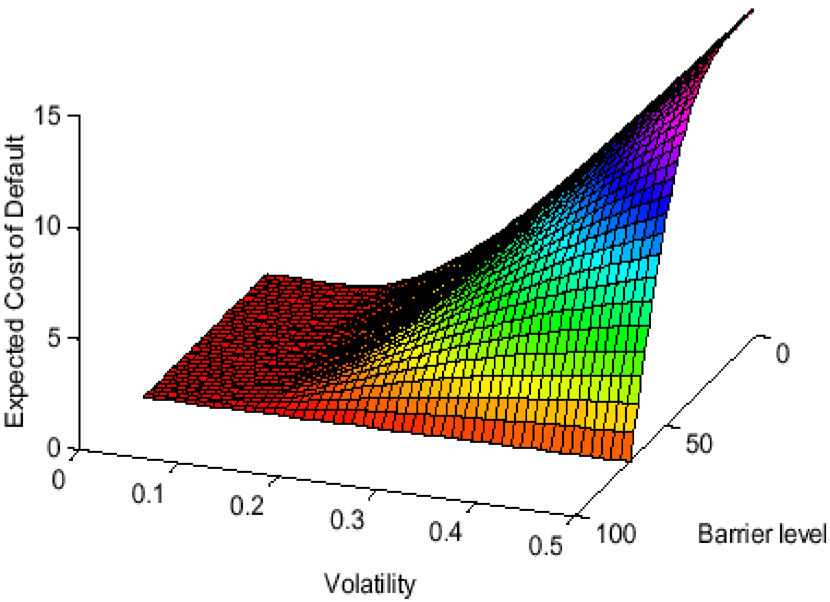


Figure 14. $V = 100$, $F = 70$, H between 2 and 70, $R = 5.43\%$, $\sqrt{T^{-}}$ between 1% and 50%, $\delta = 2\%$, $\varphi = 1$, $T = 5$.

Pirotte (1999)

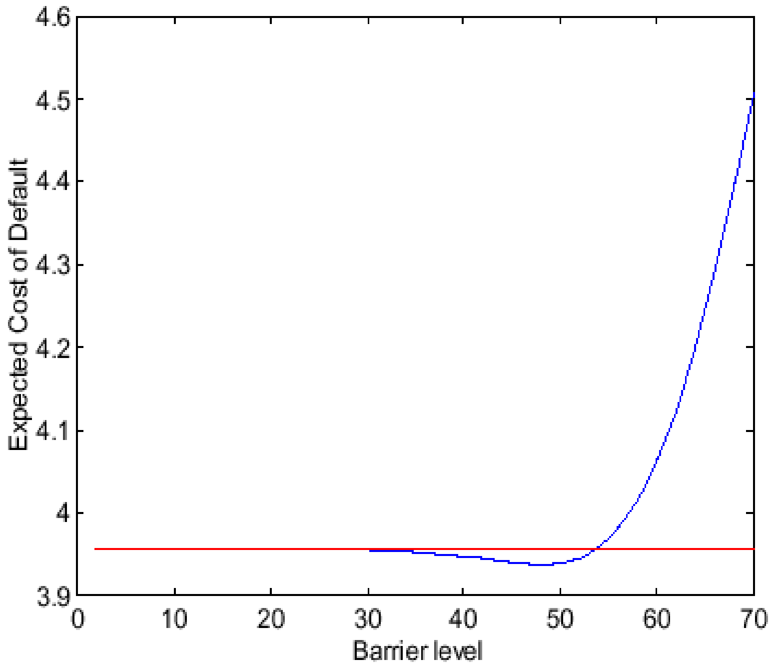


Figure 15. $V = 100$, $F = 70$, H between 2 and 70, $R = 5.43\%$, $\sqrt{T^-} = 30\%$, $\delta = 2\%$, $\varphi = 0.7$, $T = 5$.

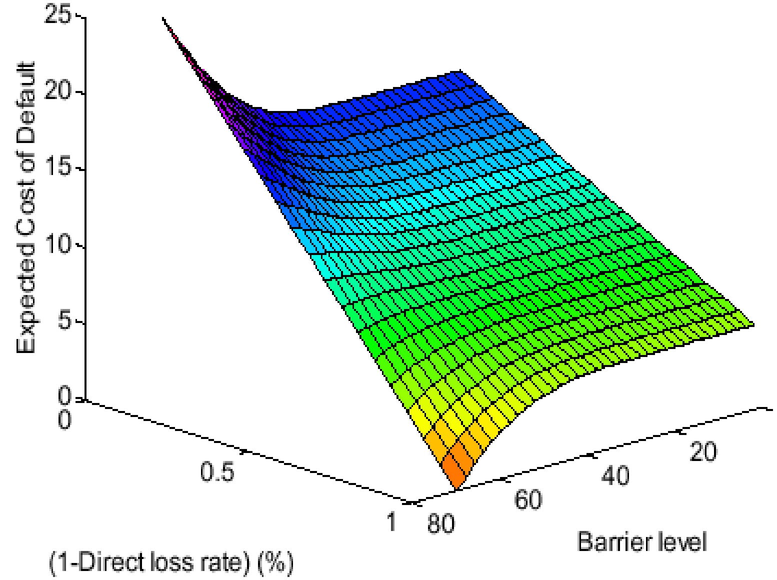


Figure 16. $V = 100$, $F = 70$, H between 2 and 70, $R = 5.43\%$, $\sqrt{T^-} = 30\%$, $\delta = 2\%$, φ between 1% and 100%, $T = 5$.

Pirotte (1999)

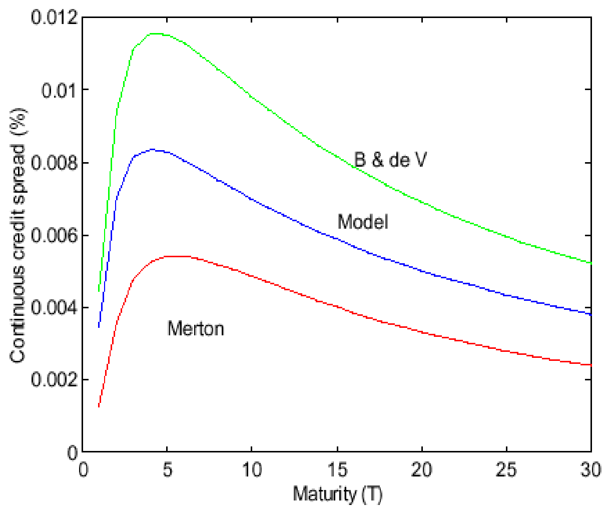


Figure 27. $V = 100, F = 60, H = 40, r$ (for Merton) $= 0.06$, liquidation costs of 0.2, $\delta = 0.02$, $\sigma_v = 25\%$. The two-factor interest rate model parameters are: $a_s = 0.5$, $a_l = 0.2$, $b_s = 0.015$, $b_l = 0.055$, $\sigma_s = 0.02$, $\sigma_l = 0.005$, $l = 0.08$, $s = 0.02$, $\rho_{sl} = 0.4$
For B&deV, Vasicek's parameters are: $a = 0.3, b = 0.06$, $\sigma_r = 0.02$,and $\rho_{vp} = 0.25$ and $\alpha = B/F$.

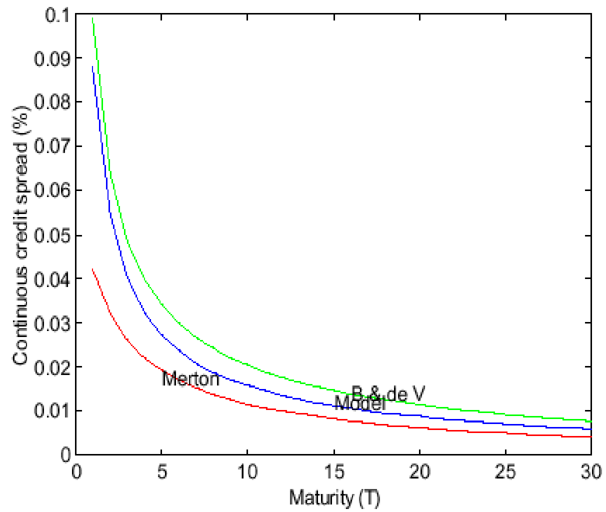


Figure 28. $V = 100, F = 90, B = 40, r$ (for Merton) $= 0.06$, liquidation costs of 0.1, $\delta = 0.8$, $\sigma_v = 25\%$. The two-factor interest rate model parameters are: $a_s = 0.5$, $a_l = 0.2$, $b_s = 0.015$, $b_l = 0.055$, $\sigma_s = 0.02$, $\sigma_l = 0.005$, $l = 0.08$, $s = 0.02$, $\rho_{sl} = 0.4$
For B&deV, Vasicek's parameters are: $a = 0.3, b = 0.06$, $\sigma_r = 0.02$,and $\rho = 0.25$ and $\alpha = B/F$.

Pirotte (1999)

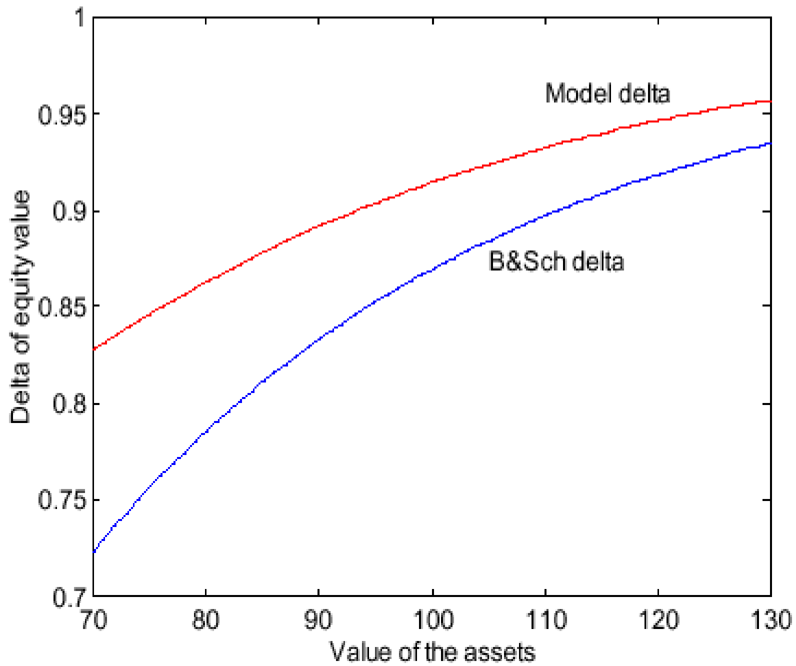


Figure 29. V between 70 and 130, $F = 70$, $B = 50$, $r = 5.43\%$, $\sigma_{-} = 30\%$, $\delta = 0\%$, $T = 5$.

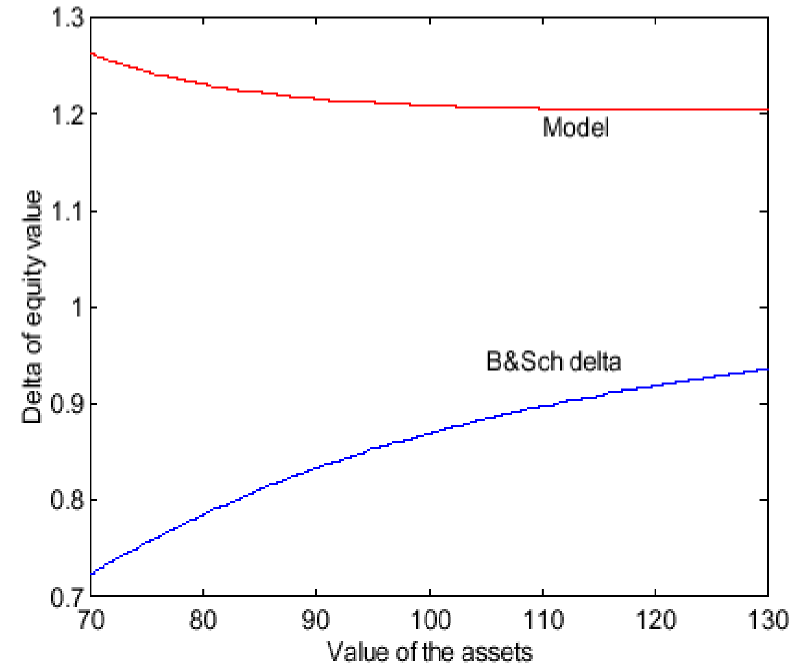


Figure 30. V between 70 and 130, $F = 70$, $B = 50$, $r = 5.43\%$, $\sigma_{-} = 30\%$, $\delta = 5\%$, $T = 5$.

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